Theoretical Physics
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Chapter A Homework. Taylor Series, Rotation Matrix, Groups

HW-A1. Matrix Multiplication. Start with the definition that

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$, where $x' = b_{11}x + b_{12}y$ and $y' = b_{21}x + b_{22}y$.

Now calculate $\begin{bmatrix} x'' \\ y'' \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix}$ using the same rule for multiplying a 2x2 matrix with a column matrix. Then arrange your result in the form

$\begin{bmatrix} x'' \\ y'' \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$ and show that you have derived the following rule for multiplying a 2x2 matrix with another 2x2 matrix:

$$\begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

HW-A2. Rotation Matrix. Show that

$$R(30°) = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}, \quad R(60°) = \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}, \text{ and } R(90°) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}.$$

Then by explicit matrix multiplications, show that

$$R(90°) = R(30°)R(60°) = R(60°)R(30°).$$

HW-A3. A Three-Element Group. Show that the set $S = \{1, -\frac{1}{2} + \frac{\sqrt{3}}{2}i, -\frac{1}{2} - \frac{\sqrt{3}}{2}i\}$ forms a group under multiplication. Construct the multiplication table. Construct a group isomorphic to this one where a person stands and undergoes rotations.