Theoretical Physics
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Chapter C Homework. Relativity and Four Vectors

HW-C1. Galilean Velocity Addition. Start with

$x' = x - vt$ and $t' = t$. Set $u' = \dfrac{\Delta x'}{\Delta t'}$ and $u = \dfrac{\Delta x}{\Delta t}$. Show $u = u' + v$.

HW-C2. Relativistic Velocity Addition. Start with

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}, \quad t' = \frac{t - x\frac{v}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{or} \quad x = \frac{x' + vt'}{\sqrt{1 - \frac{v^2}{c^2}}}, \quad t = \frac{t' + x'\frac{v}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}.$$

Set $u' = \dfrac{\Delta x'}{\Delta t'}$ and $u = \dfrac{\Delta x}{\Delta t}$. Show that $u = \dfrac{u' + v}{1 + \frac{u'v}{c^2}}$.

Then define $\beta_o = \dfrac{v}{c}$, $\beta = \dfrac{u}{c}$, and $\beta' = \dfrac{u'}{c}$ to show $\beta = \dfrac{\beta' + \beta_0}{1 + \beta'\beta_0}$.

HW-C3. Perpendicular Velocity Formula. Derive the relativistic formula for the perpendicular velocity transformation where the object moves in the K' frame with speed $\vec{u}' = (u_x', u_y')$ and the K frame measures $\vec{u} = (u_x, u_y)$. The K' frame moves at the usual speed v in the x-direction relative to the K frame. Your answer will be

$$u_y = \frac{u_y'\sqrt{1 - \frac{v^2}{c^2}}}{1 + \frac{u_x'v}{c^2}}.$$

HW-C4. Correspondence Limit. Show that a Taylor Series expansion on the first term in $KE = \dfrac{mc^2}{\sqrt{1 - v^2/c^2}} - mc^2$ will lead to the result that at slow speeds Einstein's relativistic version of the work-energy theorem reduces to Newton's: $E = \dfrac{1}{2}mv^2$.