Theoretical Physics
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Chapter F Homework. Differential Form for the Maxwell Equations

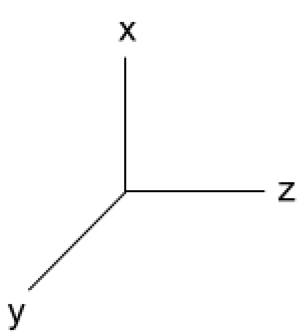
HW-F1. The Wave Equation for the Magnetic Field. Use the differential form of the Maxwell equations to derive the wave equation

$$\nabla^2 \vec{B} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$$

for the magnetic field. But instead of doing it by tediously analyzing one component like we did, take the shortcut using the following vector identity:

$$\nabla x (\nabla x \vec{A}) = \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A} \, .$$

HW-F2. The Orientation of the Electric and Magnetic Field in an E&M Wave. Start with a sinusoidal electric field vector traveling along the z axis. We will assume that the E field has a transverse and longitudinal, i.e., parallel, component. We orient the x axis so that the x axis aligns with the transverse component, i.e., the component perpendicular to the direction of propagation. Then we can write in general



$$\vec{E} = (E_x \hat{i} + E_z \hat{k}) \sin\left[k(z - ct)\right],$$

$$\vec{B} = (B_x \hat{i} + B_y \hat{j} + B_z \hat{k}) \sin\left[k(z - ct) + \delta\right],$$

where δ allows the magnetic field oscillations to be out of phase with respect to the oscillating electric fields. You will show $\delta = 0$.

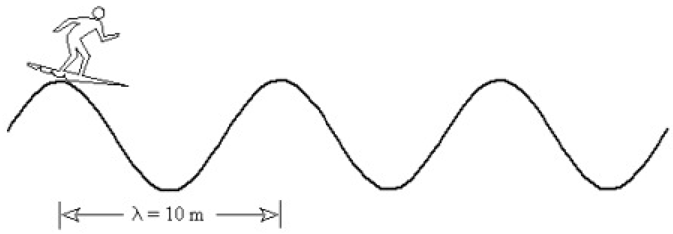
a) Use the Maxwell equation $\nabla \cdot \vec{E} = 0$ to show that $E_z = 0$.

b) Use the Maxwell equation $\nabla \cdot \vec{B} = 0$ to show that $B_z = 0$.

c) Use $\nabla x \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ to show that $B_x = 0$, $B_y = \frac{E_x}{c}$, and $\delta = 0$.

HW-F3. The Wave Relation. An ideal surfer rides on the crest of a wave.

a. If the waves are 10 meters apart and 5 crests go by second, what is the speed of the wave?



Let the distance from one crest to the next, your 10 m, be called the wavelength λ. Then let the number of crests per second be called the frequency f. Give the relation from your analysis that relates λ, f, and v (the speed of the wave). The common unit for frequency is hertz, where hertz = 1/s. Explain why this must be the case for your units to come out correctly.

b. Now consider sound waves where v = 340 m/s at room temperature and pressure. What is the wavelength in centimeters (cm) for a 440-Hz sound wave, the note they use to tune an orchestra? See the discussion on significant figures below.

c. Now consider light waves where c = 300,000 km/s, where km is kilometer, i.e., 1000 meters. What is the wavelength in cm for a 30-GHz microwave? Note that the metric prefix G stands for Giga, which is equal to one billion.

d. What is the frequency in hertz for a 645-nm red diode laser? Note that the metric prefix n stands for nano, which is equal to one billionth. Give your answer to three significant figures. Remember, never give more significant figures than the most uncertain measurement. Trailing zeros typically do not count as significant figures, but the speed of light given above is accurate to four significant figures.

Quick Review of Significant Figures and How to Handle Them

Suppose your answer from a calculation is 2.6059480 after applying some formula to a hypothetical problem. This number has 7 significant digits as it stands. The trailing zero at the end does not count, but the other zero between nonzero numbers does count. Then suppose you needed three input measured numbers for your formula and these had significant figures 3, 4, and 6 respectively. Then you should NOT report 2.6059480 as your answer since you cannot be sure of the digits beyond the third one. You should instead, round off your answer to three significant digits and report 2.61. If you include more digits in your answer because the calculator says so, then your answer is considered wrong as you cannot be sure of the accuracy of the digits beyond the third one. This is taught in intro chemistry courses and less so in intro physics courses.