Theoretical Physics
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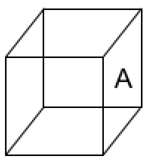
Chapter G Homework. Ideal Gas Law and Thermodynamics

HW-G1. Deriving PV = nRT for an Intro Course. Consider a gas in a cube where each particle travels at the same speed v. Also, at any given time 1/6 of the particles are moving in the +x direction, 1/6 are moving in the –x direction, 1/6 are moving in the +y direction, 1/6 are moving in the – y direction, 1/6 are moving in the +z direction, and 1/6 are moving in the –z direction.



Δl

Let N represent the total number of particles in the box.

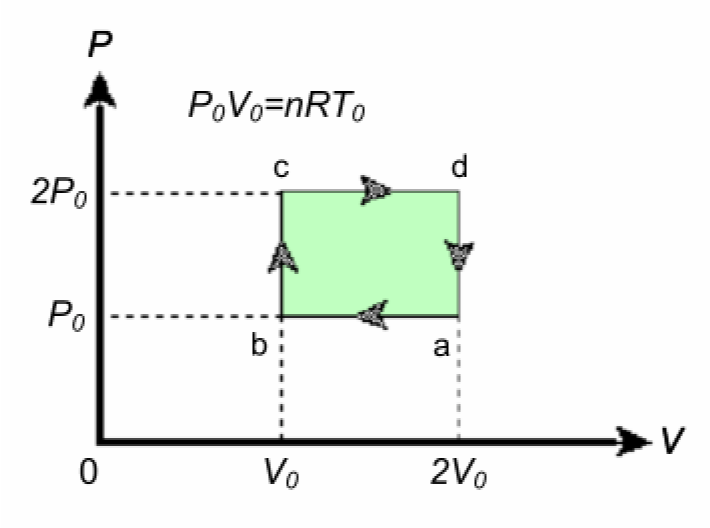
Analyze the change of momentum at the left wall and derive the ideal gas law by defining temperature in a similar way that we did in the chapter. When you get to the end of your derivation, you can use $\frac{1}{2}m\overline{v^2} = \frac{3}{2}kT$.

Now make your model more general by considering N_1 particles with velocity v_1, N_2 particles with velocity v_2, , etc. with $N = N_1 + N_2 + ...$ Keep the condition that 1/6 of the particles with velocity v_i travels along one of the axis in a given direction. What result do you get now? Hint: If you have N_1 exams with grade E_1, N_2 exams with grade E_2 and so on, what is your average grade $\overline{E}$?

HW-G2. Specific Heat at Constant Pressure. Reproduce the steps in the text in your own way to show that the specific heat at constant pressure is given by

$$c_P = c_V + R \, .$$

HW-G3. A Simple Engine. An ideal engine with an ideal gas, i.e., where $PV = nRT$, has the following cycle. **VIDEO HINT:** https://youtu.be/pZKb5BGEBfo



Compression Stroke: a to b
(compressing the gas fuel)

Ignition Phase: b to c
(with sudden pressure increase)

Expansion Phase: c to d
(engine does useful work)

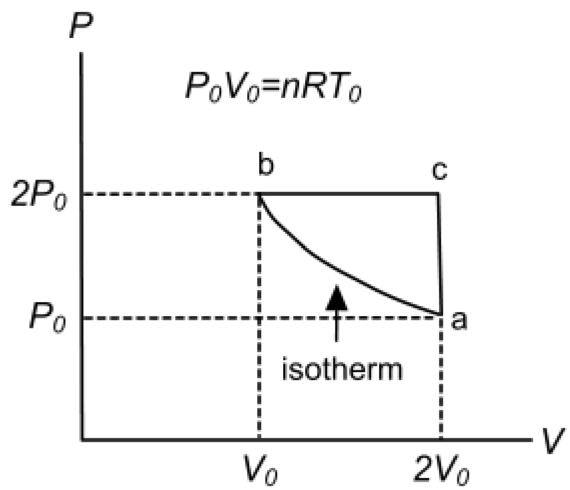
Pressure-Drop Phase: d to a
(returning to the initial PV point and ready to repeat the cycle)

Calculate the work for each of the four phases of the cycle: a-b (the isobaric compression), b-c (the isometric ignition), c-d (the isobaric expansion), and d-a (the isometric pressure drop). Then use the energy formula for an ideal gas $U = \frac{3}{2}nRT$ and the first law of thermodynamics $\Delta U = \Delta Q - P\Delta V$ to fill in the table below in terms of n, R, and T_0.

	ΔU	ΔQ	ΔW
a to b			
b to c			
c to d			
d to a			

Show that the efficiency: $\eta = \frac{W}{Q_{in}}$ is equal to $\eta = \frac{2}{13}$ for this system. Here, W stands for the net work performed and Q_{in} is the input heat (heat that flowed into the system).

HW-G4. Simple Engine With Isotherm. An engine with an ideal gas follows the a-b-c cycle shown below.



First show that the pressure and volume at the endpoints "a" and "b" satisfy the equation that describes an isothermal process for an ideal gas.

Calculate the work for each of the three phases of the cycle:

a-b (the isothermal compression),
b-c (the isobaric expansion), and
c-a (the isometric pressure drop).

Fill these in a table like the one below, all in terms of n, R, and T_0. Then complete filling in the entire table entries in terms of n, R, and T_0.

Finally, calculate the efficiency of the engine.

	ΔU	ΔQ	ΔW
a to b			
b to c			
c to a			