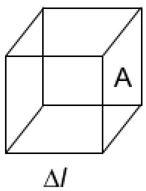
Theoretical Physics
Prof. Ruiz, UNC Asheville
Chapter G Homework. Ideal Gas Law and Thermodynamics

HW-G1. Deriving PV = nRT for an Intro Course. Consider a gas in a cube where each particle travels at the same speed v. Also, at any given time 1/6 of the particles are moving in the +x direction, 1/6 are moving in the –x direction, 1/6 are moving in the +y direction, 1/6 are moving in the – y direction, 1/6 are moving in the +z direction, and 1/6 are moving in the –z direction.



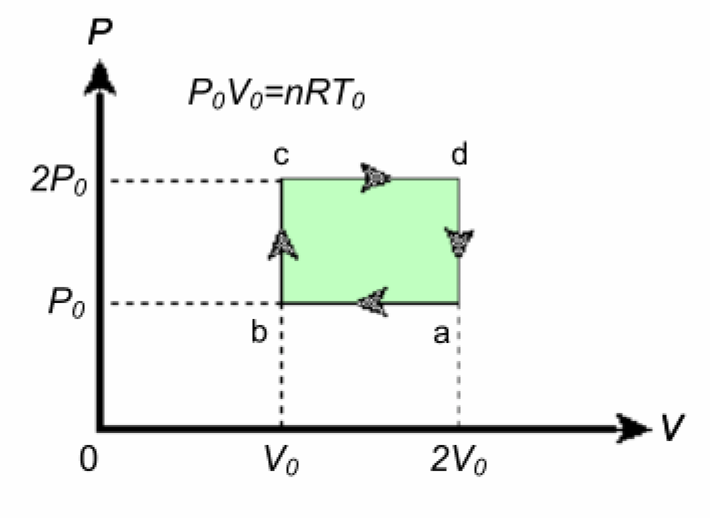
Let N represent the total number of particles in the box.

Analyze the change of momentum at the right wall and derive the ideal gas law by defining temperature in a similar way that we did in the chapter.

Now make your model more general by considering N_1 particles with velocity v_1, N_2 particles with velocity v_2, , etc. with $N = N_1 + N_2 + ...$ Keep the condition that 1/6 of the particles with velocity v_i travels along one of the axis in a given direction. What result do you get now?

Proceed to the Next Page

HW-G2. A Simple Engine. An ideal engine with an ideal gas, i.e., where $PV = nRT$, has the following cycle. **There is a VIDEO HINT for this one. See Moodle.**



Compression Stroke: a to b (compressing the gas fuel)

Ignition Phase: b to c (with sudden pressure increase)

Expansion Phase: c to d (engine does useful work)

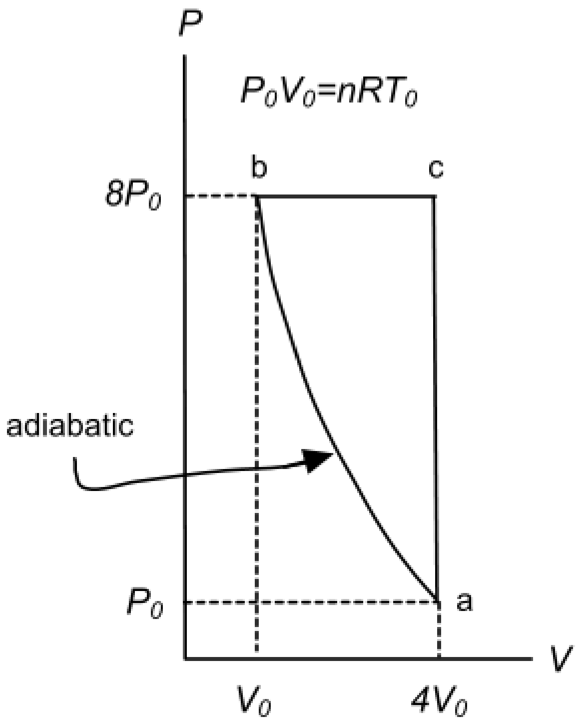
Pressure-Drop Phase: d to a (returning to the initial PV point and ready to repeat the cycle)

Calculate the work for each of the four phases of the cycle: a-b (the isobaric compression), b-c (the isometric ignition), c-d (the isobaric expansion), and d-a (the isometric pressure drop). Then use the energy formula for an ideal gas $U = \frac{3}{2}nRT$ and the first law of thermodynamics $\Delta U = \Delta Q - P\Delta V$ to fill in the table below in terms of n, R, and T_0.

	ΔU	ΔQ	ΔW
a to b			
b to c			
c to d			
d to a			

Show that the efficiency: $\eta = \frac{W}{Q_{in}}$ is equal to $\eta = \frac{2}{13}$ for this system. Here, W stands for the net work performed and Q_{in} is the input heat (heat that flowed into the system).

HW-G3. Simple Engine With Adiabatic. An engine with an ideal gas follows the a-b-c cycle shown below.



.

First show that the pressure and volume at the endpoints "a" and "b" satisfy the equation that describes an adiabatic process for an ideal gas.

Calculate the work for each of the three phases of the cycle:

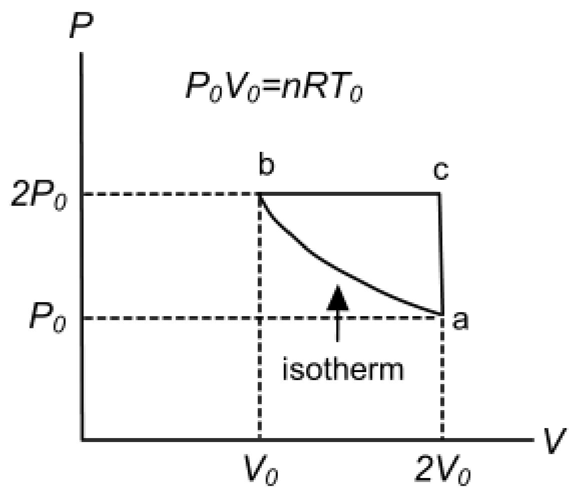
a-b (the adiabatic compression),
b-c (the isobaric expansion), and
c-a (the isometric pressure drop).

Fill these in a table like the one below, all in terms of n, R, and T_0. Then complete filling in the entire table entries in terms of n, R, and T_0.

Finally, calculate the efficiency of the engine.

	ΔU	ΔQ	ΔW
a to b			
b to c			
c to a			

HW-G4. Simple Engine With Isotherm. An engine with an ideal gas follows the a-b-c cycle shown below.



First show that the pressure and volume at the endpoints "a" and "b" satisfy the equation that describes an isothermal process for an ideal gas.

Calculate the work for each of the three phases of the cycle:

a-b (the isothermal compression),
b-c (the isobaric expansion), and
c-a (the isometric pressure drop).

Fill these in a table like the one below, all in terms of n, R, and T_0. Then complete filling in the entire table entries in terms of n, R, and T_0.

Finally, calculate the efficiency of the engine.

	ΔU	ΔQ	ΔW
a to b			
b to c			
c to a			