Theoretical Physics
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Chapter J Homework. Spinors

HW-J1. Matrix Properties. Given $\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$, where A_x, A_y, and A_z are real, (a) find the matrix $M = \vec{\sigma} \cdot \vec{A}$, where $\vec{\sigma} = \sigma_x \hat{i} + \sigma_y \hat{j} + \sigma_z \hat{k}$, and

$$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \text{ and } \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.$$

Then calculate each of the following six items:

(b) $Tr(M)$, (c) M^T, (d) M^*, (e) $M^\dagger$, (f) $\det(M)$, (g) M^{-1}.

(h) Verify that MM^{-1}.

HW-J2. Find the eigenvalues and normalized eigenvectors for $\sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$.

HW-J3. Pauli Matrices Identity. Given,

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k},$$

$$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k},$$

$$\vec{\sigma} = \sigma_x \hat{i} + \sigma_y \hat{j} + \sigma_z \hat{k},$$

show by multiplying all nine matrix combinations explicitly on the left side of the following equation, that you can regroup things to obtain the right side.

$$\vec{\sigma} \cdot \vec{A} \, \vec{\sigma} \cdot \vec{B} = \vec{A} \cdot \vec{B} \, I + i \vec{\sigma} \cdot (\vec{A} \times \vec{B}).$$