Theoretical Physics
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Chapter M Homework. The Method of Frobenius

HW-M1. Sines and Cosines. You know that the basic solutions to the differential equation

$$y''+y=0$$

are the even function $\cos x$ and the odd function $\sin x$. Use the Method of Frobenius to solve this differential equation. Find the recurrence relation. Note that your recurrence relation for the power series gives a_{k+2} in terms of a_k, which means you can break these into an even series and odd series. Since you have a second order differential equation, you have two free constants a_0 and a_1. Set $a_0=1$ and $a_1=0$ to get your even power series. See if your even power series is the same as the Taylor Series for $\cos x$. Then set $a_0=0$ and $a_1=1$ to get your odd power series. See if your odd power series is the same as the Taylor Series for $\sin x$. Work the series out to four terms for the even series and four terms for the odd series.

HW-M2. The Laguerre Differential Equation. The Laguerre differential equation is given below.

$$xy''+(1-x)y'+ny=0$$

Use the Method of Frobenius to solve this differential equation up to the point of the recurrence relation. Show that your recurrence relation for the power series terminates if n = 0, 1, 2, 3, etc.

HW-M3. Laguerre Polynomials. Use your recurrence relation from HW-M2 to find the Laguerre polynomials $L_0(x)$, $L_1(x)$, $L_2(x)$, and $L_3(x)$, where you choose $a_0=n!$ in each case.