Theoretical Physics
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Chapter N Homework. The Dirac Delta Function

HW-N1. Probability Distribution and Moments. The n^{th} central moment for the probability distribution P(x) is defined as

$$E[(x-\mu)^n] \equiv \int_{-\infty}^{\infty} (x-\mu)^n P(x)\,dx .$$

The "E" stands for expected value. Physicists like to use the term "expectation value" and use brackets. You will calculate some moments for the Gaussian centered on x = 0. Find the zeroth, first, second, third, and fourth moments for

$$P(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}} .$$

You MUST NOT LOOK UP any integrals and YOU MUST NOT USE INTEGRATION BY PARTS. Instead, from the following result, already proven in class,

$$I = \int_{-\infty}^{\infty} e^{-\alpha x^2}\,dx = \sqrt{\frac{\pi}{\alpha}} ,$$

use the derivative trick to evaluate the following two integrals, which you will need.

$$I_1 = \int_{-\infty}^{\infty} x^2 e^{-\alpha x^2}\,dx \quad \text{and} \quad I_2 = \int_{-\infty}^{\infty} x^4 e^{-\alpha x^2}\,dx$$

After you evaluate the above integrals, choose the appropriate α for your problem.

HW-N2. Dirac Delta Function. Evaluate the following two integrals, showing all steps.

$$I_{k>0} = \int_{-\infty}^{\infty} f(x)\,\delta(kx)\,dx \text{ , where } k > 0$$

$$I_{k<0} = \int_{-\infty}^{\infty} f(x)\,\delta(kx)\,dx \text{ , where } k < 0$$

Hint: Let z = kx and use what you know about the delta function from class.