Theoretical Physics
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Chapter N Homework. The Dirac Delta Function

HW-N1. Maxwell-Boltzmann Distribution: Most Probable Speed. In an earlier assignment you found the Maxwell-Boltzmann velocity distribution.

$$f(v) = 4\pi\left[\frac{m}{2\pi kT}\right]^{3/2} v^2 e^{-\frac{m}{2kT}v^2}$$

Show that the most probable speed $v_p = \sqrt{\frac{2kT}{m}}$ by setting the derivative $\frac{f(v)}{dv} = 0$ and proceeding as you would in solving a max-min problem in calculus.

HW-N2. Maxwell-Boltzmann Distribution: Average or Mean Speed. Show that the mean speed $\bar{v} = \sqrt{\frac{8kT}{\pi m}}$ by calculating from its definition $\bar{v} = \int_0^\infty vf(v)dv$. You may use an integral table to evaluate your integral.

HW-N3. Maxwell-Boltzmann Distribution: Root Mean Square Speed. Show that the root mean square speed is $v_{rms} = \sqrt{\frac{3kT}{m}}$ from its definition $v_{rms} = \sqrt{\overline{v^2}}$ where $\overline{v^2} = \int_0^\infty v^2 f(v)dv$. You may use an integral table to evaluate your integral.

HW-N4. Maxwell-Boltzmann Distribution Sketch. Sketch f(v) and sketch three dotted lines from the appropriates places on the graph down to the velocity v-axis and label each of the respective values: v_p, $\bar{v}$, and v_{rms} on the v-axis.

HW-N5. Probability Distribution and Moments. The n^th^ central moment for the probability distribution P(x) is defined as

$$E[(x-\mu)^n] \equiv \int_{-\infty}^{\infty} (x-\mu)^n P(x)dx \,.$$

The "E" stands for expected value. Physicists like to use the term "expectation value" and use brackets. You will calculate some moments for the Gaussian centered on x = 0. Find the zeroth, first, second, third, and fourth moments for

$$P(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}} \,.$$

You **MUST NOT LOOK UP** any integrals and YOU **MUST NOT USE INTEGRATION BY PARTS**. Instead, from the following result, already proven in class,

$$I = \int_{-\infty}^{\infty} e^{-\alpha x^2}\, dx = \sqrt{\frac{\pi}{\alpha}} \,,$$

use the derivative trick to evaluate the following two integrals, which you will need.

$$I_1 = \int_{-\infty}^{\infty} x^2 e^{-\alpha x^2}\, dx \quad \text{and} \quad I_2 = \int_{-\infty}^{\infty} x^4 e^{-\alpha x^2}\, dx$$

After you evaluate the above integrals, choose the appropriate α for your problem.

HW-N6. Dirac Delta Function. Evaluate the following two integrals, showing all steps.

$$I_{k>0} = \int_{-\infty}^{\infty} f(x)\delta(kx)\,dx \text{, where } k > 0$$

$$I_{k<0} = \int_{-\infty}^{\infty} f(x)\delta(kx)\,dx \text{, where } k < 0$$

Hint: Let z = kx and use what you know about the delta function from class.