Theoretical Physics
Prof. Ruiz, UNC Asheville
Chapter P Homework. Fourier Transforms

HW-P1. Fourier Transforms. Calculate the Fourier transforms of the following three functions by explicitly doing all the integrations.

a) $f(x) = e^{-ax}$ for $x \geq 0$ where $a > 0$ and $f(x) = 0$ for $x < 0$.

b) $f(x) = e^{-a|x|}$ for all x where $a > 0$.

c) $f(x) = \frac{1}{a}$ for $-\frac{a}{2} \leq x \leq \frac{a}{2}$ where $a > 0$ and $f(x) = 0$ elsewhere.

HW-P2. The Heisenberg Uncertainty Relation. The Heisenberg Uncertain Relation is $\Delta x \Delta p \geq \frac{\hbar}{2}$, where $\Delta x = \sigma_x$ is the standard deviation for the position probability distribution and $\Delta p = \sigma_p$ is the standard deviation for the momentum probability distribution. You will work with $k = \frac{p}{\hbar}$ and $\Delta x \Delta k \geq \frac{1}{2}$. The ground-state solution to the quantum-mechanical harmonic-oscillator problem. i.e., with potential $V(x) = \frac{1}{2} k_{spring} x^2$, is the Gaussian $\psi(x) = \left(\frac{\alpha}{\pi}\right)^{\frac{1}{4}} e^{-\frac{\alpha}{2}x^2}$, where $\alpha = \frac{m\omega}{\hbar}$ with $\omega = \sqrt{\frac{k_{spring}}{m}}$. The k-wave function is the Fourier transform:

$$\chi(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \psi(x) e^{-ikx} dx .$$

Calculate $\Delta x \Delta k$, i.e., $\sigma_x \sigma_k$. Explain what happens if α is small. What about large?

You know the definition of the standard deviation and you can use any integral we have done in our course. DO NOT DO ANY INTEGRALS. Use integral results from our course by simply giving the general integral with its result and applying it to your specific problem.