Theoretical Physics
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Chapter P Homework. Fourier Transforms

P1-P3. Fourier Transforms. Calculate the Fourier transforms of the following three functions by **explicitly doing all integrations.** Give the simplest form for each answer.

HW-P1. $f(x) = e^{-ax}$ for $x \geq 0$ where $a > 0$ and $f(x) = 0$ for $x < 0$.

HW-P2. $f(x) = e^{-a|x|}$ for all x where $a > 0$. Note that half of the integral, from 0 to infinity, you did in P1. Carefully do the other half and add the results.

HW-P3. $f(x) = \dfrac{1}{a}$ for $-\dfrac{a}{2} \leq x \leq \dfrac{a}{2}$ where $a > 0$ and $f(x) = 0$ elsewhere.

HW-P4. The Heisenberg Uncertainty Relation. The Heisenberg Uncertainty Relation is $\Delta x \Delta p \geq \dfrac{\hbar}{2}$, where $\Delta x = \sigma_x$ is the standard deviation for the position probability distribution and $\Delta p = \sigma_p$ is the standard deviation for the momentum probability distribution. You will work with $k = \dfrac{p}{\hbar}$ and $\Delta x \Delta k \geq \dfrac{1}{2}$. The ground-state solution to the quantum-mechanical harmonic-oscillator problem. i.e., with potential $V(x) = \dfrac{1}{2} k_{spring} x^2$, is the Gaussian $\psi(x) = \left(\dfrac{\alpha}{\pi}\right)^{\frac{1}{4}} e^{-\frac{\alpha}{2}x^2}$, where $\alpha = \dfrac{m\omega}{\hbar}$ with $\omega = \sqrt{\dfrac{k_{spring}}{m}}$. The k-wave function is the Fourier transform:

$$\chi(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \psi(x) e^{-ikx} dx .$$

The probability distributions are $P(x) = \psi^* \psi = \sqrt{\dfrac{\alpha}{\pi}} e^{-\alpha x^2}$ and $P(k) = \chi^* \chi$.

Calculate $\Delta x \Delta k$, i.e., $\sigma_x \sigma_k$. What happens to σ_x and σ_k if α increases?

You know the definition of the standard deviation from Class N. **For HW-4, DO NOT EXPLICITLY DO ANY INTEGRALS. USE INTEGRAL TABLES,** but do explain how you use the integral from the table and set the parameters to those in your specific application of the integral.