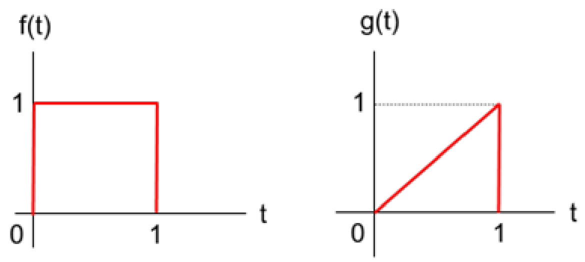
Theoretical Physics
Prof. Ruiz, UNC Asheville
Chapter Q Homework. Laplace Transforms

HW-Q1. Laplace Transform. Find the Laplace transform F(s) for the square pulse f(t) shown below by explicitly doing the Laplace transform integral. Then use the "derivative trick" for integration to obtain the Laplace transform G(s) of the ramp pulse g(t) from your result F(s) for the square pulse.



Finally, give F(1) and G(1) in terms of e, where e is the natural base.

HW-Q2. Laplace Transform Shift Property. Calculate the Laplace transform $G(s)$ for

$g(t) = t^n e^{-bt}$ two ways as described below, where $b > 0$.

a) Do the integral for the Laplace transform using the derivative trick.

b) Use the shifting property: if $g(t) = f(t)e^{\alpha t}$, then $G(s) = F(s-a)$, $s > a$.

HW-Q3. Solving a Differential Equation.

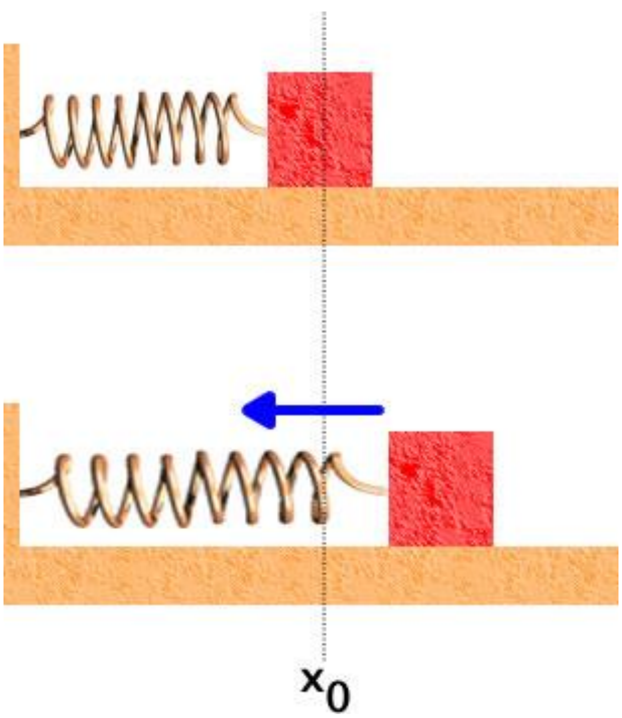


Image Courtesy David M. Harrison
Department of Physics, University of Toronto

Use Laplace transforms to solve the differential equation

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$

where you smack the block initially so that

$$x(0) = 0 \quad \text{and} \quad v(0) = A\sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}\,.$$

Simply your math using these definitions:

$$\omega_0 = \sqrt{\frac{k}{m}}\,,\ \beta = \frac{b}{2m}\,,\ \omega^2 = \omega_0^2 - \beta^2\,.$$

Hint: In the algebraic step, complete the square of the denominator before you look up the inverse Laplace Transform in the Laplace Transform Tables. You also know the answer from your Introductory Physics text, which serves as a check that you did things correctly.