Theoretical Physics
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Chapter T Homework. Poles and the Residue Theorem

T1. An Integration Along the Entire x-axis.

Use complex integration and residues to evaluate $I = \int_{-\infty}^{\infty} \frac{dx}{(x^2+1)(x^2+4)}$.

Sketch your integration path and show where the poles are in your sketch. Note that you DO NOT need to prove that the integration for your semicircle path is zero when you are doing the complex integration.

T2. An Angle Integration.

Consider the integral $I = \int_0^{2\pi} \frac{d\theta}{5+3\cos\theta}$.

Let $z = e^{i\theta}$ so that you have a circular integral path in the complex plane with unit radius. In the above given integral, get everything in terms of z. Then use the residue theorem to obtain your answer. Include a sketch of the complex plane, path of integration, and the location of the poles. Give your final answer for this integral in the most elegant form in terms of π.

Consulting the Integral Tables, you can check your answer using

$$I = \int_0^{2\pi} \frac{dx}{a+b\cos x} = \frac{2\pi}{\sqrt{a^2-b^2}}$$, where $a > b \geq 0$.