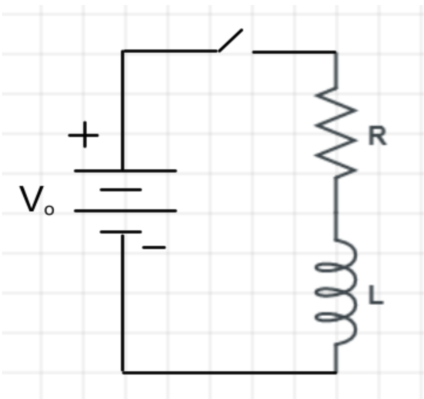
Theoretical Physics
Prof. Ruiz, UNC Asheville
Chapter U Homework. Green's Functions

U1. The Simple RL Circuit.



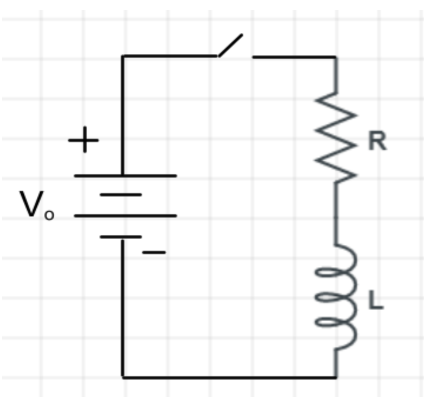
An RL circuit appears at the left. At time t = 0, the switch at the top is closed and voltage begins to be applied to the circuit. Kirchhoff's Loop Rule gives

$$V_0 - i(t)R - L\frac{di(t)}{dt} = 0 .$$

Solve this differential equation the usual way by getting the $i(t)$ and $di(t)$ on one side of the equation and the dt with constants on the other side. Integrate both side to obtain the famous result

$$i(t) = \frac{V_0}{R}(1 - e^{-\frac{R}{L}t}) .$$

U2. The Green's Function for the RL Circuit.



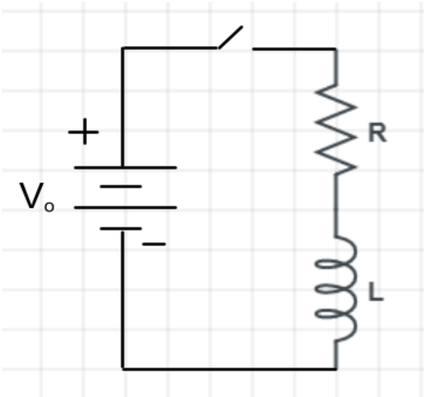
Replace the applied voltage V_0 with the impulse delta function. Then take the Fourier transform of your differential equation and solve the algebraic equation for the Fourier transform of the current $I(\omega)$. Then do the inverse Fourier transform and use complex integration to show that the Green's function is

$$G(t,0) = \frac{1}{L}e^{-\frac{R}{L}t} .$$

Then, the general soluton $i(t)$ for arbirtary input voltage $v(t)$ is given by the convolution

$$i(t) = v(t) * G(t) = \int_0^t G(t,u)v(u)du .$$

U3. The Convolution for the RL Circuit.



Now return to the applied voltage function that we had in U1. In this case, the voltage function can be written as

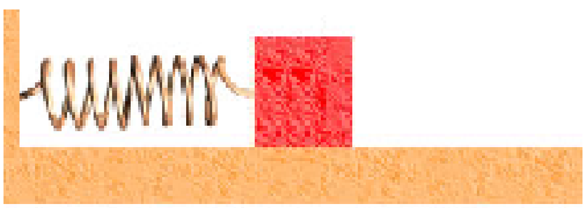
$$v(t) = 0 \text{ for } t < 0$$

$$v(t) = V_0 \text{ for } t \geq 0 .$$

Use the convolution of the applied voltage function and your Green's function to obtain the same answer you found in U1. In other words, show that

$$i(t) = \int_0^t G(t,u)v(u)du \quad \text{leads to} \quad i(t) = \frac{V_0}{R}(1 - e^{-\frac{R}{L}t}) .$$

U4 and U5. The Green's Function for the Damped Harmonic Oscillator.



Courtesy David M. Harrison
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The differential equation for the damped harmonic oscillator is

$$\frac{d^2x}{dt^2} + 2\beta\frac{dx}{dt} + \omega_0^2 x = 0$$, which comes from $F = -kx - bv = ma$ with

$\omega_0^2 = \dfrac{k}{m}$ and $\beta = \dfrac{b}{2m}$. For your specific problem $\alpha^2 = \omega_0^2 - \beta^2 > 0$.

Use our procedure (delta function, Fourier transform, inverse Fourier transform with complex integration, Green's function) to show that the Green's function for the damped harmonic oscillator system is given by the following function encountered in introductory physics.

$$G(t,0) = \frac{1}{\alpha}e^{-\beta t}\sin(\alpha t)$$