**Theoretical Physics** **Exam 1** **Spring 2020**
**Prof. Ruiz, UNC Asheville** **Study Guide**

## Class A. Taylor Series, Groups.

**1. Taylor Series.** Know how to use Taylor Series Expansions. It is best to memorize the following expansions for small x for the functions listed in PA1 (Practice Problem 1), especially since you have seen these before in math and physics classes. They are used so often in courses.

$$\cos x \approx 1 \qquad \sin x \approx x \qquad \tan x \approx x \qquad e^x \approx 1 + x$$

**2. Matrices.** Know how to multiply matrices, e.g.,

$$\begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}\begin{bmatrix} 0 & i \\ i & 4 \end{bmatrix} = \begin{bmatrix} 1\cdot 0 + 2i & 1\cdot i + 2\cdot 4 \\ 3\cdot 0 + 0\cdot i & 3\cdot i + 0\cdot 4 \end{bmatrix} = \begin{bmatrix} 2i & 8+i \\ 0 & 3i \end{bmatrix}.$$

See PA4 for more sample exam type questions.

**3. Groups.** Know the definition of a group: binary operation with closure, association, identity element, and inverse. PA6, PA7, PA8, and PA9 are good sample exam type questions.

## Class B. Euler's Formula and Integral Tricks.

**1. Power Series.** You should memorize the following two very important power series, especially since you have encountered these before in math or physics classes.

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$$

$$(a+b)^n = a^n b^0 + \frac{n}{1} a^{n-1} b^1 + \frac{n(n-1)}{1\cdot 2} a^{n-2} b^2 + \frac{n(n-1)(n-2)}{1\cdot 2\cdot 3} a^{n-3} b^3 + \ldots$$

What patterns make these easy to remember?

**2. Euler Formula.** You also should memorize the Euler formula: $e^{i\theta} = \cos\theta + i\sin\theta$, seen before in math classes. Sample Exam Type Questions are PB2 and PB3 (short and sweet problems).

**3. Integral Tricks.** Know the 4 integral tricks : Real-Imaginary, Polar, Derivative, and Complete the Square. Good exam type questions include PB4 and PB5.

## Class C. Relativity.

**1. Galilean Transformation.** Know how to derive the simple Galilean Transformation:

$$x' = x - vt \text{ and } t' = t.$$

**2. Lorentz Transformation**. If Modern Physics had been a prerequisite for this course, I would include the Lorentz Transformation in the memorized listing. But since Modern Physics was not required, no need to memorize the Lorentz Transformation.

A sample exam type question from the work we did deriving the Lorentz transformation would be: suppose $\tan\theta = i\frac{v}{c}$, what is $\cos\theta$ ? You then set up the cute triangle as we did in class and found in the text. Use the Pythagorean Theorem to find the hypotenuse without worrying about the imaginary number to get $\sqrt{1-\frac{v^2}{c^2}}$ . Then you find $\cos\theta = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ . What about $\sin\theta$ ? Combining the material with the Taylor Series material you need to know from Class B above, can you show the following for small $\frac{v^2}{c^2}$ without looking anything up?

$$\cos\theta = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \approx 1 + \frac{1}{2}\frac{v^2}{c^2}$$

Hint: Use your memorized formula for the binomial expansion with $a = 1$, $b = -\frac{v^2}{c^2}$, and $n = -\frac{1}{2}$.

**3. Relativistic Space and Time**. Sample questions would be using the Lorentz Contraction or Time Dilation if you were given these formulas and then asked to do something with them involving basic math manipulations.

**4. Relativistic Velocity Addition.** A sample question would be, if given the Lorentz Transformation, can you take deltas and do the divisions like in homework to obtain relativistic velocity additions. Sample exam type questions are PC1 (nonrelativistic), PC2, and PC3.

**5. The Energy Triangle.** Know the cute triangle that has E on the hypotenuse, pc on the vertical side, and $mc^2$ on the horizontal side so that the Pythagorean Theorem gets you $E^2 = m^2c^4 + p^2c^2$.

**Class D. "Derivation of the Maxwell Equations." There will be nothing on the exam from this chapter.**

**Class E. Differential Form for the Maxwell Equations.** For this chapter, focus on Section E4, the use of the del Operator. Know how to calculate the gradient, divergence and curl. The Practice Problems PE1, PE2, PE3, and PE4 are excellent sample exam type questions.

**Class F. "Let There Be Light."**

**1. The Wave Equation.** Memorize the differential form of wave equation.

$$\frac{\partial^2\psi(x,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2\psi(x,t)}{\partial t^2}$$

Hint: Remember the dimensional reasoning to assist you in memorization. Also, know that the general solutions are $\psi(x,t) = Af(x-vt) + Bg(x+vt)$. Once you have the above, you can get the three-dimensional form easily

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2}$$, which is the same as $$\nabla^2\psi = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2}.$$

**2. Calculating Divergences and Curls.** This technique is really already covered in Chapter E. So here you see more of the same thing. For a sample question, here is a good one. How about taking $\vec{E} = x^2 y\,\hat{i}$ and show that the identify $\nabla\times(\nabla\times\vec{E}) = \nabla(\nabla\cdot\vec{E}) - \nabla^2\vec{E}$ is true for this specific $\vec{E}$. You do the left side first by calculating $\nabla\times\vec{E}$. The answer is $\nabla\times\vec{E} = -x^2\,\hat{k}$. Then you continue to show that $\nabla \times (\nabla \times \vec{E}) = 2x\,\hat{j}$. See if you can do all the necessary work to arrive at these results. Then you turn to the right side and see if you get the same answer. First you calculate $\nabla \cdot \vec{E}$. Can you show that $\nabla \cdot \vec{E} = 2xy$? Then see if you can show that $\nabla(\nabla \cdot \vec{E}) = \nabla(2xy) = 2y\,\hat{i} + 2x\,\hat{j}$. What about $-\nabla^2 \vec{E}$ on the right side? See if you can show that $-\nabla^2 \vec{E} = -2y\,\hat{i}$. Finally, when you combine your right hand pieces, does the result agree with the left side?

**Class G. Ideal Gas Law and Thermodynamics.** For this chapter, let's wait until the second exam where we will combine this Chapter G with Chapter H, which is also on thermodynamics.