

1. Matrices. $[\sigma_x, \sigma_y] = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} - \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$$[\sigma_x, \sigma_y] = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix} - \begin{bmatrix} -i & 0 \\ 0 & i \end{bmatrix} = \begin{bmatrix} 2i & 0 \\ 0 & -2i \end{bmatrix} = 2i \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = 2i\sigma_z.$$

2. Integral.

$$\int_{-\infty}^{\infty} \frac{x \sin x}{x^2 + 1} dx = \lim_{m \rightarrow 1} \left[-\frac{d}{dm} \right] \left[\int_{-\infty}^{\infty} \frac{\cos mx}{x^2 + 1} dx \right] = \lim_{m \rightarrow 1} \left[-\frac{d}{dm} \right] \left[\pi e^{-m} \right]$$

$$= \lim_{m \rightarrow 1} \left[\pi e^{-m} \right] = \pi e^{-1} = \pi / e \quad \text{or}$$

$$\lim_{\substack{m \rightarrow 1 \\ a \rightarrow 1}} \left[-\frac{d}{dm} \right] \left[\int_{-\infty}^{\infty} \frac{\cos mx}{x^2 + a^2} dx \right] = \lim_{\substack{m \rightarrow 1 \\ a \rightarrow 1}} \left[-\frac{d}{dm} \right] \left[\frac{\pi}{a} e^{-ma} \right] = \lim_{\substack{m \rightarrow 1 \\ a \rightarrow 1}} \left[\frac{\pi}{a} a e^{-ma} \right] = \frac{\pi}{e}$$

3. Relativity.

$$L_x = L_x' \sqrt{1 - \frac{v^2}{c^2}}, \quad L_y = L_y', \quad \tan \phi' = L_y' / L_x'$$

$$\tan \phi = \frac{L_y}{L_x} = \frac{L_y'}{L_x' \sqrt{1 - \frac{v^2}{c^2}}} = \frac{\tan \phi'}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{For } \phi' = 30^\circ \text{ and } v = \frac{2\sqrt{2}}{3}c$$

$$\tan \phi = \frac{(1/2) / (\sqrt{3}/2)}{\sqrt{1 - \left[\frac{2\sqrt{2}}{3} \right]^2}} = \frac{1/\sqrt{3}}{\sqrt{1 - \frac{8}{9}}} = \frac{1/\sqrt{3}}{\sqrt{\frac{1}{9}}} = \frac{3}{\sqrt{3}} = \sqrt{3} \quad \text{so } \phi = 60^\circ$$

4. Vector Calculus. $\vec{A} = xz \hat{i}$ and. $\nabla \times \vec{A} = \partial \vec{B} / \partial x$

$$\nabla \times \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xz & 0 & 0 \end{vmatrix} = x \hat{j} = \partial \vec{B} / \partial x$$

$$\vec{B} = \int x dx = \frac{x^2}{2} + f(y, z)$$

$$\vec{B} = \left[\frac{x^2}{2} + f(y, z) \right] \hat{j} + g(y, z) \hat{i} + h(y, z) \hat{k}$$