Prescribing Eyeglasses for Myopia and Hyperopia

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Most eyeglass prescriptions are given for patients with one of two common visual problems: myopia and hyperopia. Myopia is the condition where the eye cannot clearly focus on far objects; e.g., one can't easily see the blackboard from the back of the room. Hyperopia refers to problems seeing close up, e.g., difficulty reading the newspaper. Physics enables us to estimate the prescription of eyeglasses quickly from data anyone can gather. The beauty of the method derives from the fact that you do not need to know anything about the detailed structure of the eye's compound lens system and biological media. This is due to the fact that eyeglasses are corrective.

Although introductory physics texts treat this subject,[1,2] it is only for the myopic case that the analysis is so easy that it can be virtually performed in one's head. Here we present a way whereby the hyperopic case can also be quickly determined. The student needs to know the definition of diopters, the fact that diopters add when thin lenses are placed together, and basic ray-tracing rules for thin lenses.

The normal eye can see distant mountains. We say that the far point is infinity. We emphasize that the focal length or associated dioptric power[3] of the eye necessary to see the distant mountains is irrelevant. Consider a myopic eye with a far point $d_{far} = 2$ m. Anything beyond this 2-m distance is blurred. However, the eye can see close up, i.e., it is nearsighted (sighted to see near).

Here is where the usual approach is helpful. We need to bring those distant mountains to the myopic eye's far point. By choosing a diverging lens where $f = -d_{far}$, we allow parallel rays from infinity to form a virtual image at the far point. The eye doctor prescribes the lens in diopters, the reciprocal of the focal length where the focal length is expressed in meters. The prescription is therefore $D = -1/d_{far}$, where d_{far} is in meters and the minus sign indicates a diverging lens. For $d_{far} = 2$ m, the prescription is $D = -0.5$ diopter. To estimate a prescription for a myopic eye, find the farthest distance where observed objects such as posters are in fairly sharp focus. This gives an estimate of the far point and the prescription.

For a hyperopic eye, we determine how far a book must be placed from the eye in order for our patient to read comfortably. This is the hyperopic eye's near point. The normal near point is taken to be 25 cm, i.e., 1/4 m, a comfortable reading distance. If one has to hold a book farther away, then that person is hyperopic, which is sometimes called farsighted. The eye is sighted to see far and cannot see nearby objects well.

When the normal eye focuses on print 1/4 m away, it produces an extra 4 diopters. Here is how to arrive at this important fact. The base dioptric strength of the eye occurs when the eye focuses at infinity. The simplest optical model of the eye places this value at 60 diopters,[4] but we do not need to know this. Light enters the eye parallel to the optic axis in such a case. The eye then focuses these rays on the retina. Here is our trick. To focus on an object 1/4 m away, break down what the eye has to do into two steps.[5] First, we need those rays that diverge from the point 25 cm away to become parallel, and second, these parallel rays must finally be focused on the retina.

To get the rays to be parallel takes 4 diopters, which can readily be seen by sketching a converging lens

 DOI: 10.1119/1.1855743

1/4 m away from the point and making the outgoing rays parallel. Then, the base dioptric power of the eye does the rest to focus the rays. The eye of course is more complicated, but it must do the equivalent of these very steps we reason out from the physics. Once again, the details of refraction by the cornea and eye lens are not needed for our analysis.

Now consider someone who needs to hold a newspaper 1/3 m away instead of the normal 1/4 m. This near point of 1/3 m means that the eye can produce an additional 3 diopters of power. Now we see why eye doctors like diopters. When you combine lenses, the diopters add. So the prescription is simply 1 diopter, in order to enable our hyperopic eye achieve the desired 4 diopters to read at the normal distance. Think of our hyperopic eye as being 1 diopter short of the needed extra 4 diopters.

Here is how you can determine a prescription by borrowing someone's glasses. You can easily measure the focal length of a converging lens by imaging distant trees on a sheet of paper. This takes care of hyperopic cases. Clever ways of measuring the focal length of a diverging lens have appeared in the literature.[6] Below is a quick nonmathematical method using parallax. Have someone hold the diverging lens in front of you as you look through it to see small virtual images of distant objects. If you move your head left and right, there will be parallax—the small images will shift relative to other objects in your peripheral view. A third person places a pencil below the lens and between the lens location and the distant objects. The distance of the pencil from the lens is then adjusted according to your directions until the pencil (outside of the field of view seen through the lens) appears to move with the virtual images of distant objects (seen through the lens). When this occurs, you have the focal length; it is the distance from the lens to the pencil (with a minus sign since the lens is diverging). From this measurement, you can arrive at an estimate of the prescription in diopters.

Finally, here is a formula to formalize our analysis and to make a connection with the usual relations in texts. For the hyperopic eye, we can summarize our steps by writing

$$\frac{1}{f_{\text{prescription}}} = \frac{1}{d_{\text{normal near}}} - \frac{1}{d_{\text{near point}}},$$

where the normal near point is 25 cm, corresponding to 4 diopters. The $1/f$ is the prescription in diopters. You can see that you need to prescribe a positive focal-length lens since the hyperopic near point is greater than the normal near point. If your near point should be less than 25 cm, then you are definitely OK on the near end; i.e., there is no need for glasses to read a book. The above formula is actually the same type of formula for our myopic eye, where we can write

$$\frac{1}{f_{\text{prescription}}} = \frac{1}{d_{\text{normal far}}} - \frac{1}{d_{\text{far point}}},$$

taking the normal far point to be infinity. You can readily see the prescription will be for a negative focal-length lens since the far point of the myopic eye is less than infinity. The above formulas can be obtained directly from the traditional lens equation by choosing appropriate object and image distances with the proper sign conventions.

References

1. Eugene Hecht, *Physics: Algebra/Trig,* 3rd ed. (Brooks/Cole, Pacific Grove, CA, 2003), pp. 865–868.
2. James S. Walker, *Physics*, 2nd ed. (Prentice-Hall, Upper Saddle River, NJ, 2004), pp. 892–898.
3. The dioptric power of a lens is simply the strength of the lens in diopters, i.e., the reciprocal of the focal length when the focal length is expressed in meters.
4. Michael P. Keating, *Geometrical, Physical, and Visual Optics*, 2nd ed. (Butterworth-Heinemann, Boston, 2002), p. 75.
5. David S. Falk, Dieter R. Brill, and David G. Stork, *Seeing the Light: Optics in Nature, Photography, Color, Vision, and Holography* (Harper & Row, New York, 1986), pp. 159–160 (now published by Wiley).
6. Scott C. Dudley, "How to quickly estimate the focal length of a diverging lens," *Phys. Teach.* **37**, 94 (Feb. 1999); David L. Wagner and Thomas A. Walkiewicz, "When the eye meets the lens," *Phys. Teach*. **38**, 474 (Nov. 2000).

PACS codes: 01.40K, 01.55, 42.66

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