A Black Hole in Our Galactic Center

Michael J. Ruiz, University of North Carolina at Asheville, Asheville, NC

An introductory approach to black holes is presented along with astronomical observational data pertaining to the presence of a supermassive black hole at the center of our galaxy. Concepts of conservation of energy and Kepler's third law are employed so students can apply formulas from their physics class to determine the mass of the black hole that resides in the center of the Milky Way.

We first derive the formula that gives the condition for a black hole from conservation of energy. Students encounter the conservation of energy formula where the kinetic energy plus potential energy at one point is set equal to that for a second point: A common application is to find the escape velocity v from a celestial body of mass M and radius R where atmospheric friction is neglected. We send a projectile of mass m with speed v from the surface of our large body so that the projectile comes to rest at infinity. The conservation of energy equation then gives:

$$\frac{1}{2}mv^2 - \frac{GMm}{R} = 0 + 0,$$

where we use the gravitational potential energy

$$U(r) = -\frac{GMm}{r}$$

at $r = R$ (the start of the journey) and $r = \infty$ (the end of the trip).

Long ago John Michell (1784) and Pierre-Simon Laplace (1796) applied Newtonian concepts to investigate a theoretical celestial body where the escape velocity would be such that light could not escape. Since Newtonian gravitation (1687) and Einstein's general theory of relativity (1915) are different frameworks, one must be cautious in making analogies between classical situations and black holes of general relativity.[1] But one cannot resist setting $v = c$ for the escape velocity and arriving at[2,3]

$$R = \frac{2GM}{c^2}.$$

This relation turns out to be the condition for a black hole in general relativity.[4] The parameter R is called the Schwarzschild radius after Karl Schwarzschild, who discovered the first solution to Einstein's field equations shortly after Einstein proposed the theory of general relativity.[5]

If mass M is concentrated into a sphere with radius $R = \frac{2GM}{c^2}$, the Schwarzschild radius, a black hole forms. Forces become so great that the mass evolves to a point of infinite density called the singularity. However, we expect this central region to be finite when we eventually arrive at a theory that combines general relativity with quantum mechanics.

A spacecraft can orbit the black hole outside the Schwarzschild radius, but should the spacecraft come closer than this special radius, the spacecraft cannot escape. The spacecraft mass then joins the mass of the black hole, and the Schwarzschild radius becomes a trifle larger as a result. Students can calculate the Schwarzschild radius for the Sun and Earth. If one could somehow get all the mass of the Sun into a volume defined by the Schwarzschild radius, you would produce a black hole. This can lead to discussions of

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supernovae, natural phenomena whereby black holes form when the most massive of stars die.

Astronomers believe that supermassive black holes lurk in the centers of many galaxies, including our own.[6,7] These extremely large black holes formed ages ago over billions of years as they "ate" matter over long periods of time, growing larger and larger. In recent years, near-infrared studies of stars close to the center of the Milky Way have revealed data from which we can calculate the mass of our central galactic black hole. We will show below how this is accomplished with basic physics, giving introductory students a problem from the frontiers of astronomical research.

R. Schödel et al.[8] have found that the star S2 (S0-2 in Ref. 11) orbits the center of our galaxy with a semi-major axis of about 1000 AU. It comes as close as 120 AU to the galactic center and has an incredibly short orbital period of only 15 years. For a quick comparison, the distance from the Sun to Pluto is 40 AU, and Pluto orbits the Sun once every 250 years. Kepler's third law enables us to measure the mass of the black hole[9] at the center of our galaxy.

Kepler's third law[10] for an object of negligible mass *m* orbiting a massive body *M* is

$$\frac{GM}{4\pi^2} = \frac{a^3}{P^2},$$

where the semi-major axis is *a* and the period *P*. Dividing our equation by a similar expression for the Sun with mass M_0, we obtain

$$\frac{M}{M_o} = \frac{(a/a_o)^3}{(P/P_o)^2}.$$

The simplest units are found by setting $a_0 = 1$ AU and $P_0 = 1$ year so that *a* is then expressed in AU and *P* in years. Note that when $M = M_0$, we obtain the familiar Kepler form for the solar system in Earth units.[11]

Using $a = 1000$ AU and $P = 15$ years, we obtain for the mass of our black hole

$$M = \frac{1000^3}{15^2} M_0 = 4 \times 10^6 M_0,$$

i.e., 4 million solar masses!

Table I contains data for three stars from Ghez et al.,[12] where Schödel's S2 is included as S0-2. Students can find an estimate of the supermassive black hole for these cases and even employ a spreadsheet. Four more stars, with larger uncertainties, are given in Ref. 12.

Table I. Some Stellar Orbital Parameters from Ghez et al.

Star	Semi-major Axis *a* (AU)	Period *P* (yr)	Closest Approach in AU
S0-2	919 ± 23	14.53 ± 0.65	122.2 ± 2.7
S0-16	1680 ± 510	36 ± 17	45 ± 16
S0-19	1720 ± 110	37.3 ± 3.8	287 ± 25

Students should calculate the Schwarzschild radius for a supermassive black hole consisting of 4 million solar masses. How does this compare to 120 AU, the closest approach of S0-2? What about the closest approach made by S0-16? It is also interesting to express these distances in light hours. Students can place an upper observational bound on the spatial volume within which the central mass must reside. A comparison can be made with the concentration of matter in our solar system. Matter confined to a relatively small volume of space enhances the case for a black hole. By considering such questions, students are engaged with the same issues arising from observational data that concern the professional astronomer.[13]

The European Southern Observatory (ESO) has an excellent website[14] with much information about S2. One can view a nice image of the orbit of S2 around the central black hole in the Milky Way as well as a video clip describing its motion. There is also a fairly recent NOVA program, "Monster of the Milky Way," on the massive black hole in the center of our galaxy. The video of this program can be obtained on DVD.[15]

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References

1. A. K. Raychaudhuri, "Black holes and Newtonian physics," *Gen. Relat. Gravit.* **24**, 281–283 (March 1992) and

J. Eisenstaedt, "Dark body and black hole horizons," *Gen. Relat. Gravit.* **23**, 75–80 (Jan. 1991).

2. R. W. Brehme, "On the radius of black holes," *Am. J. Phys.* **42**, 1039 (Nov. 1974).
3. Many years ago (1979) at a UNC-Chapel Hill workshop, Hendrik van Dam derived this formula for a group of visiting introductory teachers. He facetiously stated that we used the "wrong formulas" (i.e., nonrelativistic) for kinetic energy and potential energy, and our two mistakes canceled to get the correct general-relativistic result!
4. Technically in general relativity, one must interpret the *r*-coordinate as that computed from the circumference of a sphere surrounding the center. Due to the curvature of space, the directly measured radial distance between two nested spheres is larger than the difference found using the general-relativistic *r*-coordinate. See E. F. Taylor and J. A. Wheeler, *Exploring Black Holes: Introduction to General Relativity* (Addison-Wesley Longman, San Francisco, 2000), Chap. 2, pp. 7-11, 28.
5. For a nice historical treatment of black holes for the general reader, see K.S. Thorne, *Black Holes & Time Warps: Einstein's Outrageous Legacy* (W. W. Norton & Company, New York, 1994).
6. J.M. Pasachoff and A. Filippenko, *The Cosmos: Astronomy in the New Millennium,* 3rd ed. (Thomson Brooks/Cole, Belmont, CA, 2007).
7. M.A. Seeds, *Stars and Galaxies*, 5th ed. (Thomson Brooks/Cole, Belmont, CA, 2007).
8. R. Schödel et al., "A Star in a 15.2-year orbit around the supermassive black hole at the centre of the Milky Way," *Nature* **419**, 694–696 (Oct. 17, 2002).
9. M. Zeilik and S.A. Gregory, *Introductory Astronomy & Astrophysics*, 4th ed. (Saunders College Publishing, Fort Worth, TX, 1998), p. 346. A situation where Kepler's third law needs to be modified is a binary system of black holes orbiting near each other. A nice diagram showing deviations from Kepler's third law in such an extreme case is found in the otherwise advanced paper P. Grandclément, E. Gourgoulhon, and S. Bonazzola, "Binary black holes in circular orbits. II. Numerical methods and first results," *Phys. Rev. D* **65**, 044021-12 (2002).
10. For the more general case where the smaller mass is not negligible, see p. 16 of Ref. 9 for a derivation of

$$\frac{G(M+m)}{4\pi^2} = \frac{a^3}{P^2}.$$

11. M.J. Ruiz, "Kepler's third law without a calculator," *Phys. Teach.* **42**, 530–532 (Dec. 2004).
12. A.M. Ghez et al., "Stellar orbits around the galactic center black hole," *Astrophys. J.* **620,** 744–757 (2005). The leading author, Andrea Ghez, has published a book and DVD for grade school students to inspire women to become astronomers: A.M. Ghez and J.L. Cohen, *You Can Be a Woman Astronomer* (Cascade Pass, Culver City, CA, 2006).
13. Readers are encouraged to consult the introductory texts of Refs. 6 and 7 for other observations relating to the supermassive black hole in the center of the Milky Way, including historical perspectives.
14. http://www.eso.org/outreach/press-rel/pr-2002/pr-17-02.html.
15. http://www.pbs.org/wgbh/nova/blackhole/.

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Michael J. Ruiz, *professor of physics at UNCA, specializes in teaching general education courses in light, sound, and astronomy. His innovative e-texts for these courses have been featured on CNN.*

Department of Physics, University of North Carolina at Asheville, Asheville, NC 28804; ruiz@unca.edu