Color Addition and Subtraction Apps

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Color addition and subtraction apps in HTML5 have been developed for students as an online hands-on experience so that they can more easily master principles introduced through traditional classroom demonstrations. The evolution of the additive RGB color model is traced through the early IBM color adapters so that students can proceed step by step in understanding mathematical representations of RGB color. Finally, color addition and subtraction are presented for the X11 colors from web design to illustrate yet another real-life application of color mixing.

The apps introduced here were developed[1] for our liberal arts optics course for non-science majors at UNC Asheville. This course has no prerequisites, which means these apps can easily be used in high school. Readers can access the online apps and videos or download them. Note that two of the apps require a web server. All educational, non-profit use is permitted.

Demonstrations illustrating color mixing by addition and subtraction have always taken a prominent place in *The Physics Teacher.* Creative ideas for demonstrations have appeared in Ron Edge's "String and Sticky Tape Experiments" column,[2] Joe Pizzo's "Deck the Halls,"[3] Walter Connolly's,[4] Karl C. Mamola's,[5] and Erlend H. Graf's[6] "Apparatus for Teaching Physics," and in the "Trick of the Trade" section.[7] These publications are in addition to notes and articles on color mixing.[8-14] The apps presented here serve as complementary online activities for students to further explore and strengthen their understanding of color mixing.

RGB (color addition) and CMY (color subtraction) models

Limitations of the models

This article is restricted to RGB color models (using the additive primaries red, green, and blue) and CMY color models (using the subtractive primaries cyan, magenta, and yellow) for producing colors. In each case we obtain a range of colors referred to as a color space or gamut, which represents a subset of the total amount of colors the human eye can perceive. The complete color space for human perception is described with the *International Commission on Illumination* (CIE) color space. The reader can consult texts such as *Seeing the Light* by Falk, Brill, and Stork[15] or *Light Science: Physics and the Visual Arts*[16] by Rossing and Chiaverina for a discussion of color gamuts (subspaces) and the associated horseshoe-shaped chromaticity diagram of the larger CIE color space.

The classic color demonstrations

The classic demonstration of color mixing by addition involves projecting red, green, and blue colored beams to a wall in a darkened room using three projectors or cleverly adapting one for the purpose.[14] When the beams overlap, the colors from each beam combine and reflect from the wall to our eyes. The colors therefore mix additively.

The classic demonstration of color mixing by subtraction involves overlapping cyan, magenta, and yellow filters on an overhead projector. White light must pass through one or more filters. Since a filter absorbs light, it subtracts some of the light from the original source. Therefore, we have color mixing by subtraction.

Note that with addition we start with black: a dark room. We then mix color by adding light as we overlap beam regions. The combined light on the wall then reflects to our eyes. With color subtraction we start with white light and "subtract" from the white light as it passes through filters. Refer to Fig. 1.

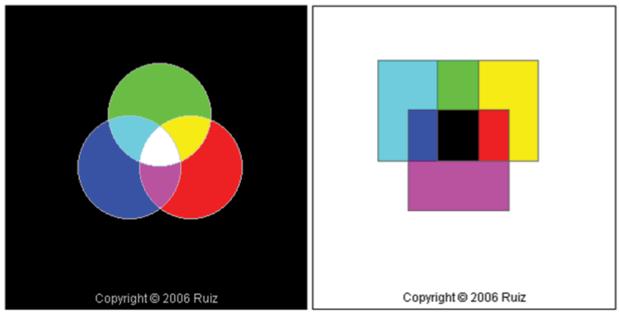

Fig. 1. Still frame image of color addition and subtraction apps. View the complete video at *TPT Online,* http://dx.doi.org/10.1119/1.4928344_1. Explore the interactive apps for color addition and subtraction at http://www.mjtruiz.com/TPT/color/fig-01-beams-filters/colorAddition.html and http://www.mjtruiz.com/TPT/color/fig-01-beams-filters/colorSubtraction.html.

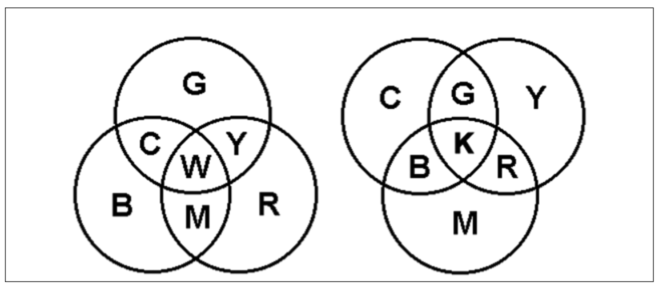

Fig. 2. Additive (left) and subtractive (right) rules.

There is a nice symmetry evident from the above arrangement of overlaps. A student at the University of Maryland pointed this symmetry out to one of the authors (MJR) when he was a graduate teaching assistant. You can obtain the subtractive rules by "turning the additive rules inside out." You take the inside C-Y-M pattern found in the additive mixture and move it to the outside for the subtractive pattern. Then you take the outside G-B-R pattern from the additive mixture and place it in the inside of the subtractive pattern. Finally, you replace white (W) with black (K) when going from the additive to the subtractive pictures. This relationship is also illustrated in Fig. 2. My students like this observation as they can memorize the additive rules and then use the trick to quickly arrive at the subtractive rules.

Idealized primaries and complementary colors

The simple broadband model for primary color spectral power distributions[15,16] provides ideal graphs to approximate the colors used in these demonstrations. Always remember that the colors in this paper are broadband colors and not spectral or monochromatic colors (colors with single wavelengths like laser light). Broadband colors are also called nonspectral colors since more than one spectral color is present in their composition. See Fig. 3 for our idealized broadband colors.

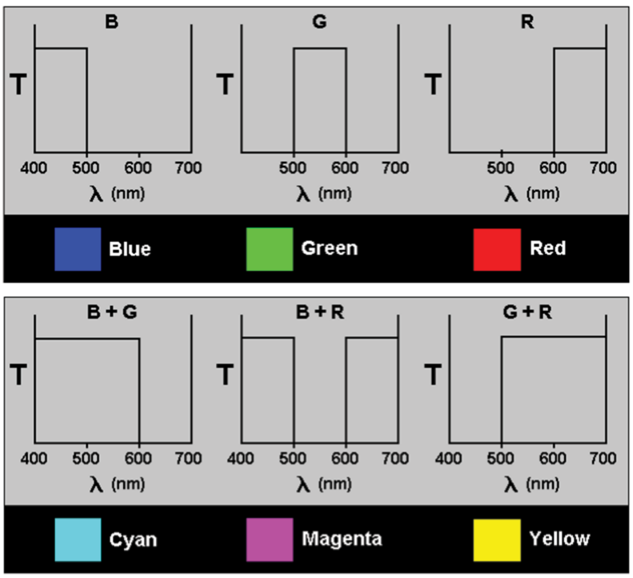

Fig. 3. Idealized graphs of transmittance versus wavelength.[15]

When discussing the rules of subtractive color mixing illustrated in Fig. 2, some students will be confused since they will state from art class that mixing blue and yellow makes green. Technically this is not true since the artist's blue is really a cyan. If you consider our above ideal blue and yellow, there is no green in common because the rectangular-shaped blue and yellow spectra do not overlap. With no overlap, there is no resulting transmitted light. Therefore, a subtractive mixture of these two produces black. The artist Michael Wilcox states this conclusion in the title of his book on subtractive color mixing with paints: *Blue and Yellow Don't Make Green.*[17] Just as no overlap in a Venn diagram indicates the null set, no overlap in spectra for subtractive color mixing results in black.

Therefore, spectral graphs come to the rescue to clear up the confusion that arises[10] when the subtractive primaries cyan, magenta, and yellow are referred to as blue, red, and yellow in art classes. From the ideal graphs we see that cyan is made up of blue and green while broadband yellow consists of green and red. When you overlap a cyan and a yellow filter, the common green can pass through both. Therefore, a subtractive mixture of cyan and yellow produces green.

Similarly, filters appearing bluish, such as those made by Rosco for theater lights,[9] will transmit some green. I (MJR) always refer to such bluish filters as cyan filters to steer away from the confusion described above. The spectral graphs clarify these subtle subtractive color-mixing effects. For another subtle example with filters, see Keeports[8] where the author analyzes the transmission spectra for a particular set of blue and amber sunglasses with the surprising result that, in this case, yellow and blue make red!

Another possible source of confusion from art class is complementary colors. One can define complementary colors within the context of color addition: two colors are defined to be complementary if the two colors mixed additively produce white. Therefore, the complementary pairs are blue-yellow, green-magenta, and red-cyan. One can think in terms of thirds of the spectrum. Consider blue. Our broadband blue consists of the first third of the spectrum in our idealized model, while our broadband yellow consists of the second two thirds. Adding these gives us all three thirds of the spectrum, i.e., white.

In this way each additive primary can be united with its complementary subtractive primary to complete itself with all thirds of the spectrum, which is white. One can also define complementary colors within the context of color subtraction. In that case, the complementary color for a color filter is the color that the filter absorbs. A blue filter absorbs yellow, i.e., subtracts yellow out from white. Therefore, blue and yellow are complementary pairs. Overlapping filters made of complementary pairs subtractively produce black.

Mixing various amounts of primaries

A more detailed demonstration that gives deeper understanding of color mixing is one that allows us to control the strengths of each primary. This level of control is difficult to achieve for additive mixing in the classroom since it requires dimmer controls or awkwardly moving one of the light sources farther or closer to the screen to decrease or increase its brightness on the wall. For color subtraction it is even more difficult because the filter density has to change. With our next pair of apps, we can easily vary the strengths of the primaries in order to make subtle variations in color for both color addition and color subtraction. A screen shot of color mixing with different strengths of additive and subtractive primaries is shown in Fig. 4.

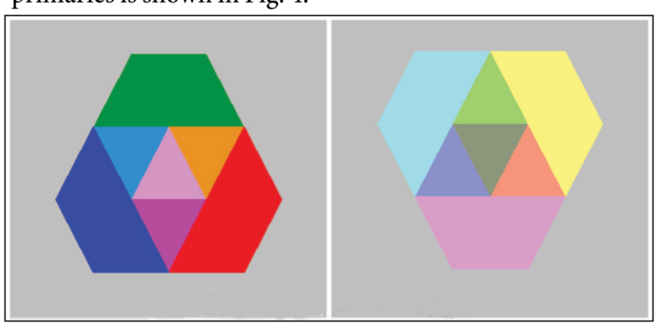

Fig. 4. Still frame images from the apps used for controlling the amounts of primary colors. Watch the video at *TPT Online,* http://dx.doi.org/10.1119/1.4928344_2 and explore the app, http://www.mjtruiz.com/TPT/color/fig-04-primary-amounts/strengthPrimaryAddSub.html.

Fig. 5. (a) Still frame images of flowers with additive primaries. Watch the video at *TPT Online*, http://x.doi.org/10.1119/1.4928344_3. Explore the app at http://www.mjtruiz.com/TPT/color/fig-5a-flowers-add/flowers-addition.html. (b) Flowers with subtractive properties. Watch the video at *TPT Online*, http://dx.doi.org/10.1119/1.4928344_4. Explore the app at http://www. mjtruiz.com/TPT/color/fig-5b-flowers-sub/flowers-subtraction.html.

Notice how we can obtain an orange in our additive mixture by adding lots of red, less green, and no blue. See the orange triangle in the left image in Fig. 4 where green overlaps red and there is no blue overlap. To subtractively make orange, we can filter out blue, filter out some green, and let the red pass. Thus the yellow filter needs to be strongest, followed by magenta, with no cyan filter. See the orange triangle in the right image where yellow and magenta overlap in the absence of cyan.

Such fine tuning allows us to reproduce colorful scenes from everyday life, though within the limitation of our RGB color space. In Fig. 5(a) we show the additive primary components needed to produce a photo taken by the author's (MJR) wife, Margaret, in 2002. Think of addition as projecting these images in a dark room and overlapping the red, green, and blue images.

In Fig. 5(b) we show the subtractive primary components needed to produce the same image of Fig. 5(a). Think of subtraction as placing these images on an overhead and overlapping the three cyan, magenta, and yellow images.

The evolution of RGB computer color

Three-bit RGB color

Many students enter college these days knowing something about HTML for web pages and color descriptions such as background-color: #00FF00 or background-color: rgb(0,255,0). They may even have designed a few web pages. A nice way to achieve a deep understanding of what these HTML color descriptions mean is to trace the development of color for the personal computer, which started with the simplest RGB scheme. The engineers first had to master making eight basic colors before they could eventually design a device to display over 16 million.

The simplest RGB color system allows each of the primaries to only take on two values: off or on. Let off be represented by 0 and on by 1 (full strength for this simple system). The eight combinations we encountered for the additive rules in Fig. 1 can be represented in Table I. The scheme utilizes three "slots" called bits, i.e., a single bit, 0 or 1, for each of the three primaries. The results of our additive color-mixing rules now take on a mathematical description, a binary one (base 2).

Table I. The three-bit RGB model.

RGB	000	001	010	011	100	101	110	111
Name	black	blue	green	cyan	red	magenta	yellow	white
Color								

A computer monitor, or television, consists of numerous tiny red, green, and blue pixels packed very close to each other. In our simple scheme described above, each pixel can be either off or on. Their close proximity allows for color mixing in our eye as the colors virtually emanate from the same small region on the monitor. This type of additive mixture is called partitive mixing and is similar to the color addition of closely spaced dots in a Seurat painting.

The first IBM color adapter for the PC (1981), the Color Graphics Adapter (GCA), used this simplest of schemes in order to obtain eight basic colors for displaying text.[18] Figure 6 illustrates the three-bit scheme where magenta is produced as a result of turning on the blue and red pixels: RGB = 101.

Six-bit RGB color

In 1984 IBM brought its Enhanced Graphics Adapter (EGA) to the market. This graphics card allowed for each of the three additive primaries to be in one of four levels of brightness: off, low, medium, and high. We need two bits to represent these four states: 00 = off, 01 = low, 10 = medium, and 11 = high. Therefore, EGA is a six-bit color system with a total of 4 x 4 x 4 = 64 colors.[19] Figure 6 has a link to illustrate six-bit color.

Twenty-four-bit RGB color

The public was not satisfied until the technology advanced to a 24-bit color system (c. 1990), where each primary color is represented by eight bits. This allows for each primary to take on $2^8 = 256$ shades. The total amount of colors is 256 x 256 x 256 = 16.8 million to three significant digits. See the link in Fig. 6 for a 24-bit color example.

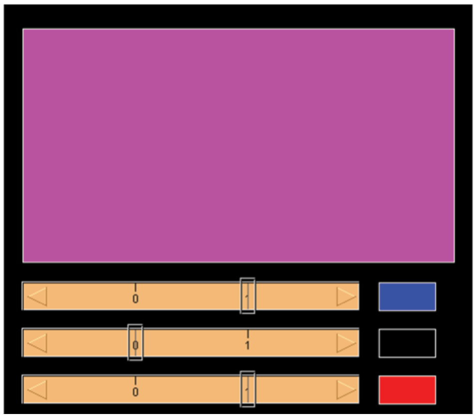

Fig. 6. Still frame image of magenta pallette in the three-bit RGB color scheme. Watch the instructional video for the apps at *TPT Online*, http://dx.doi.org/10.1119/1.4928344_5. Explore the app at http://www.mjtruiz.com/TPT/color/fig-06-color/RGB-03-bit.html. Watch the six-bit and 24-bit instructional videos at *TPT Online*, http://dx.doi.org/10.1119/1.4928344_6 and http://dx.doi.org/10.1119/1.4928344_7. Explore the six-bit and 24-bit apps at http://www.mjtruiz.com/TPT/color/fig-06-color/RGB-06-bit.html and http://www.mjtruiz.com/TPT/color/fig-06-color/RGB-24-bit.html.

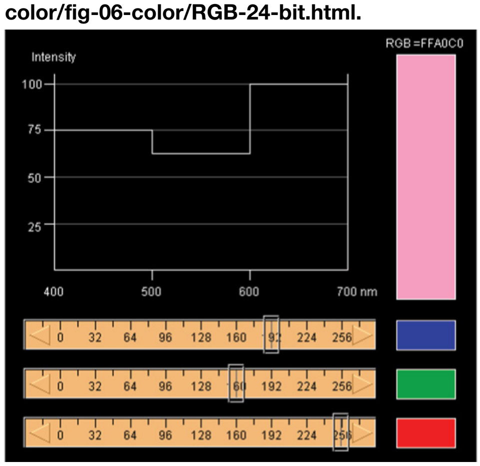

Fig. 7. Still frame image of a lavender tint with hexadecimal value RGB = FFA0C0. Watch the instructional video for the app at *TPT Online*, http://dx.doi.org/10.1119/1.4928344_8. Explore the app at http://www.mjtruiz.com/TPT/color/fig-07-hex/hexadecimal.html.

The 24-bit scheme lends itself easily to the hexadecimal system, i.e., base 16. Each primary ranges from decimal value 0 to 255, which in hexadecimal is 00 to FF. The digits in hexadecimal are the decimal numbers 0 through 9 and then the additional letters A, B, C, D, E, and F for decimal values 10 through 15. The hex value FF means 15 x 16 + 15 = 255 as we are in base 16. This method is the common way web designers set colors in HTML. Figure 7 shows a lavender with red at 100% strength, i.e., 255 decimal, green at 60 decimal, and blue at 92 decimal. The hexadecimal equivalent is given above the color swatch: RGB = FFA0C0.

Art students may get confused when they see "intensity" since in art this term refers to the purity or saturation of the color. To avoid confusion, I often just call the vertical axis "amount" or "percent" in class when I sketch such graphs on the board.

The X11 color set

As a final treat, we turn to the X11 color set, which consists of 140 colors with defined RGB values along with common names. This color set dates back to the early 1990s and is widely used in web design.[20] While some students may appreciate binary and hexadecimal, practically everyone perks up in response to such colorful names as Alice Blue, Forest Green, Fuchsia, Lavender Blush, Lemon Chiffon, Khaki, and Hot Pink. Isn't physics cool?

These colors with their interesting common names add another dimension to the physics of color mixing. It activates another part of the brain and brings another real-life application to the student due to its widespread use by web designers. Note that there are a few oddities such as aqua and cyan having the same RGB and dark gray being lighter than gray.

Let's analyze dark orange from the X11 color set, shown in Fig. 8. The red is at the maximum value 255 decimal (FF hexadecimal), green is at 140 decimal (8C hex), and there is no blue. If desired, you could take the base-16 hex value of 8C, which means eight 16s and C units to arrive at 8 x 16 + 12 = 140, which is the decimal value for our green. Note the strength for the complementary filter (subtractive mixing) is given by subtracting the respective additive primary value from 255. Since red is at maximum 255, the complementary color filter cyan is 255 – 255 = 0, i.e., fully diluted and transparent in order to let all the red through. A cyan filter at full strength would absorb all of our red and we do not want that. So we turn the cyan filter off by fully diluting it to a clear transparent sheet. Refer to Fig. 8 where you can see that the "cyan" filter appears white on the overhead since its strength is 0.

The magenta filter is set at 255 – 140 = 115. Remember that 255 for a filter means full strength and 0 means fully diluted so that it is completely transparent. Finally, since there is no blue in the additive mixture, we want a yellow filter since yellow absorbs blue, its complement. To allow absolutely no blue through for the subtractive mixing, the yellow filter is at full strength, i.e., at 255.

In summary, the respective sums of the complementary pairs always give 255. You can consider the green for our dark orange, which is set at 140, as a partial green. What is missing is the difference 255 – 140 = 115. Therefore, in the subtractive scenario we set the magenta filter to 115 so that the magenta filter subtracts out this 115 amount of green and leaves the desired 140 to pass through the filter. Our cyan filter set at 0 absorbs no red, and our yellow filter set at 255 absorbs all the blue. These principles apply to all color-mixing situations, whether the specific color under investigation is in the X11 color set or not.

More interdisciplinary connections and conclusion

The interactive color apps we have seen in this paper are relevant to a wide variety of disciplines. The additive primaries red, green, and blue are used by engineers to produce colors on our computer and television monitors. We have seen that the numerical representation of the RGB values involves mathematical bases: binary, decimal, and hexadecimal. Subtractive color mixing is fundamental in art and color printing. The typical four-color printing process uses the subtractive primaries cyan, magenta, and yellow, along with a black ink.

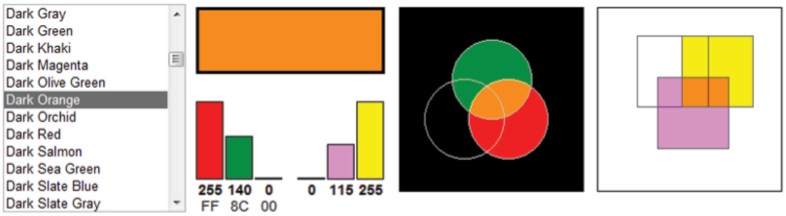

Fig. 8. Still frame image of a dark orange tint from the X11 color set. Watch the video at *TPT Online,* http://dx.doi.org/10.1119/1.4928344_9. Explore the app at http://www.mjtruiz.com/TPT/color/fig-08-x11/X11ColorSet.html.

Color photography (traditional or digital) employs the recording of RGB information (with RGB layers in film or RGB sensors). A color photo that you can put in your scrapbook uses the subtractive filters or dyes over a white paper canvas. The now-antiquated Kodachrome slide (1935-2009) consisted of cyan, magenta, and yellow layers to filter white light into the colorful reproduction of the original scene. A knowledge of color theory is necessary for color correction of images with photo-editing software. Web designers think in terms of RGB for writing HTML and CSS (Cascading Style Sheets) code. Web-related code such as "background-color: #00FF00" or "background-color: rgb(0,255,0)" takes on new meaning for the students. They immediately recognize additive color mixing, which is pervasive everywhere on web pages throughout the world.

Students can be encouraged to think of other real-life applications of color mixing. The interactive color-mixing activities included in this paper allow students to master color physics as they actively engage with the online HTML5 modules. The theory comes to life. Most importantly, they have fun as they learn.

References

1. Computer apps discussed in this paper were originally developed as Java applets by Evan M. Ruiz through partial funding from the University of North Carolina General Administration, UNC Asheville, and the Cisco Learning Institute. Evan's sister Frances Ruiz redesigned the apps in HTML5 for this paper.
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17. Michael Wilcox, *Blue and Yellow Don't Make Green, Revised Edition* (School of Color Publishing Ltd., Bristol, UK, 2001).
18. The basic eight CGA colors could be boosted in intensity to obtain a brighter version of the basic set. The CGA adapter had several video modes, some for text and some for graphics. The full 16-color palette could be used in the text modes, and in graphics modes only four colors of the complete set could be used simultaneously.
19. The EGA graphics card had various video modes displaying at most a 16-color palette simultaneously from its 64-color table.
20. Thomas A. Powell, *HTML & CSS: The Complete Reference*, 5th ed. (McGraw-Hill, New York, 2010). See Appendix C: Colors.

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