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Covariant harmonic oscillators and the axial-vector coupling constant

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The SU(6) value for the axial-vector coupling constant of the nucleon, which is 5/3, is modified by considering the relativistic motion of the quarks inside the hadron. The covariant harmonic-oscillator formalism of Kim and Noz is used to describe the internal quark motion. An excellent numerical result is obtained with reasonable choices for spring constant and quark mass.

The SU(6) quark model prediction of $\frac{5}{3}$ for the axial-vector coupling constant is somewhat higher than the experimental value of 1.2. Several attempts have been made to account for the discrepancy by considering the internal motion of the quarks.[1,2] Using the Dirac equation and treating the quarks independently in central potentials, Bogoliubov[3] has shown that the correction to the SU(6) result is related to the expectation value of the orbital angular momenta of the quarks. Le Yaouanc *et al.*[4] have shown that free Dirac spinors, with a nonrelativistic harmonic-oscillator momentum distribution describing the binding, give an encouraging result for $-g_A/g_V$. However, they have made the approximation $E+m=2m$ in the small components of the Dirac spinors. Therefore, their calculation is not completely covariant.

In this note we use the covariant harmonic-oscillator model of Kim and Noz.[5] The strength of their model lies in the fact that the inner product in momentum space is defined as a covariant integral over energy and momentum. Furthermore, there exists a probability interpretation for their covariant harmonic-oscillator wave functions. Therefore, unlike all previous attempts to modify the SU(6) result, the present treatment is fully covariant.

Kim and Noz[5] have constructed a complete set of normalizable solutions for the covariant harmonic-oscillator equation of Feynman *et al.*[6] The solutions satisfy a very simple orthogonality relation[7] which enables us to give a Lorentz-contracted probability interpretation for these Kim-Noz wave functions.[8] Furthermore, the orthogonality relation has the correct correspondence limit with nonrelativistic quantum mechanics.

The wave function for a three-quark system with external momentum P is

$$\psi(\xi,k,q)=\delta(P-\xi)\chi(P,k)\chi(P,q), \qquad (1)$$

where k and q are the two independent internal momenta and $\chi(P,k)$ is the ground-state harmonic-oscillator wave function. In the center-of-momentum frame $\chi(P,k)$ becomes

$$\chi(k)=\frac{1}{\Omega\pi}\exp\left[-\frac{1}{2\Omega}(k_1^2+k_2^2+k_3^2+k_0^2)\right]. \qquad (2)$$

For the matrix elements of the weak current between nucleon states we take[8]

$$\left\langle N\right|\sum_{\text{cyclic}}\sum_{\text{spins}}\int u(p_1)\bar{u}(p_1)u(p_2)\bar{u}(p_2)u(p_3)\bar{u}(p_3)\gamma_\mu(1+\gamma_5)u(p_3)\bar{u}(p_3)\left|\psi(p_1,p_2,p_3)\right|^2 d^4p_1d^4p_2d^4p_3\tau_+^{(3)}\left|N\right\rangle, \qquad (3a)$$

where the spin sums are for $i=1,2$,

$$\sum_{\text{spins}}u(p_i)\bar{u}(p_i)=\frac{\gamma\cdot p_i+m_q}{2m_q} \qquad (3b)$$

and for $i=3$,

$$\sum_{\text{spins}}u(p_3)\bar{u}(p_3)\gamma_\mu(1+\gamma_5)u(p_3)\bar{u}(p_3)=\frac{\gamma\cdot p_3+m_q}{2m_q}\gamma_\mu(1+\gamma_5)\frac{\gamma\cdot p_3+m_q}{2m_q}. \qquad (3c)$$

The nucleon state $|N\rangle$ is the isospinor $\binom{\psi_p}{\psi_n}$. In the center-of-momentum frame, ψ_p and ψ_n are the usual SU(6) wave functions of the proton and neutron, respectively. Figure 1 illustrates the content of Eq. (3). The Dirac spinors $\bar{u}(p_i)$ and $u(p_i)$ represent the creation and destruction of quarks which reach relativistic internal velocities. The wave function $\psi(p_1,p_2,p_3)$, which becomes $\chi(q)\chi(k)$ using Eq. (1), determines the momentum distribution of the binding. The spins of the quarks are properly symmetrized with the unitary spin quantum numbers in the SU(6) wave function. We sum over the spins of the Dirac spinors which experience the binding. For the quarks we take the vector and axial-vector coupling constants to be equal.

We can write Eq. (3) as

$$\langle N|j_\mu|N\rangle=\frac{1}{(2m_q)^4}\left\langle N\left|\sum_{\text{cylic}}\int(\gamma\cdot p_1+m_q)_{(1)}(\gamma\cdot p_2+m_q)_{(2)}\left[(\gamma\cdot p_3+m_q)\gamma_\mu(1+\gamma_5)(\gamma\cdot p_3+m_q)\right]_{(3)}|\psi(p_1,p_2,p_3)|^2\right.\right.$$
$$\left.\left.\times d^4p_1d^4p_2d^4p_3\tau_+^{(3)}\right|N\right\rangle. \qquad (4)$$

The above expression is covariant and no approximations have been made. We evaluate Eq. (4) in the center-of-momentum frame. The nonzero contributions to the vector and axial-vector matrix elements are $\mu=0$ and $\mu=3$, respectively. We choose the conjugate variables used by Feynman *et al.*[6] The renormalized axial-vector coupling constant is

$$-\frac{g_A}{g_V}=\frac{\langle N|A_3|N\rangle}{\langle N|V_0|N\rangle}=\frac{F_A(x)\left\langle N\left|\sum_{i=1}^{3}\sigma_z^{(i)}\tau_+^{(i)}\right|N\right\rangle}{F_V(x)\left\langle N\left|\sum_{i=1}^{3}\tau_+^{(i)}\right|N\right\rangle}\,. \qquad (5)$$

The quantity x is defined as Ω/m^2, where $m=\frac{1}{2}(m_q+\frac{1}{3}M_0)$. The quark mass is m_q and the nucleon mass is M_0. When the momentum integrals are performed, we find

$$-\frac{g_A}{g_V}=\tfrac{5}{3}[1-\delta(x)]\,, \qquad (6)$$

where

$$\delta(x)=\frac{\frac{8}{9}x-\frac{1}{162}x^2}{16+\frac{1}{3}x-\frac{1}{216}x^2}\,. \qquad (7)$$

When $x\to 0$, which corresponds to ignoring internal motion, $\delta\to 0$ and Eq. (6) reduces to $\frac{5}{3}$.

In Fig. 2 we plot $-g_A/g_V$ vs quark mass m_q for several values of Ω. The traditional value of Ω, which is 1.05 GeV^2, is determined from the slope of the Regge trajectories. However, this value does not accommodate the higher-mass resonances very well. In particular the Roper resonance, which has $N=2$ and $L=0$ in the harmonic-oscillator scheme, is an exceptionally low-mass resonance.

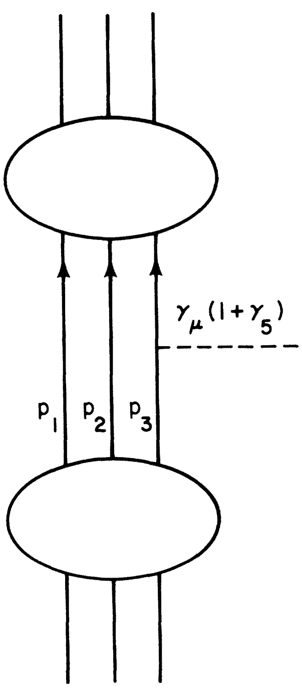

FIG. 1. Structure of the internal quarks interacting with an external weak current. These quarks are described by covariant harmonic-oscillator wave functions.

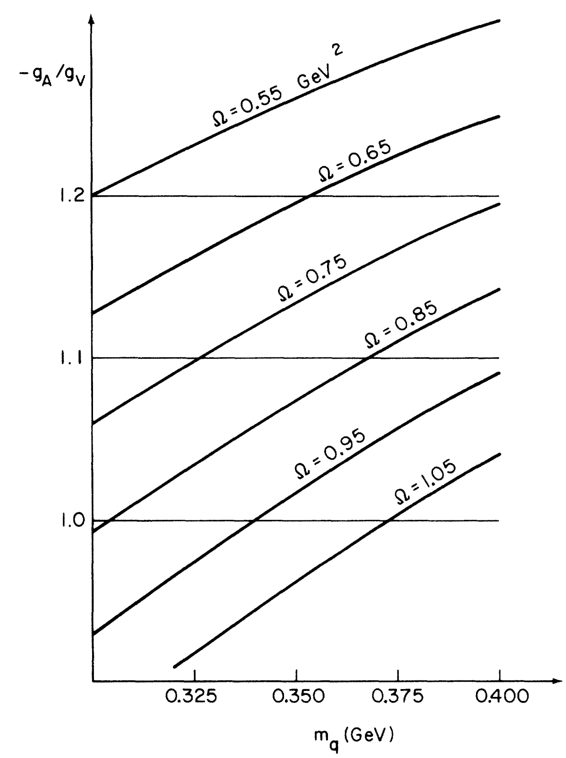

FIG. 2. Plot of $-g_A/g_V$ vs m_q for several values of Ω. Kim and Noz (Refs. 9 and 10) suggest that Ω lies between 0.65 and 0.75 GeV^2. A choice for Ω within these limits gives a very desirable result for $-g_A/g_V$.

Recently Kim and Noz[9,10] studied all the nonstrange baryon resonances up to 2000 MeV within the framework of the harmonic-oscillator model. Their mass formula gives good results for all the resonances including the controversial Roper resonance. According to their formula the linear mass spacing is 270 MeV. This strongly indicates that Ω lies between 0.65 and 0.75 GeV^2. In this range we find an excellent result for the axial-vector coupling constant in the neighborhood of acceptable quark-mass values.

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