Students Dance Longitudinal Standing Waves

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Abstract

A demonstration is presented that involves students dancing longitudinal standing waves. The resulting kinaesthetic experience and visualization both contribute towards an understanding of the natural modes of vibrations in open and closed pipes. A video of this fun classroom activity is provided: Ruiz M J, Video: Longitudinal Dance, 2017, http://mjtruiz.com/ped/dance/.

The dance

The wave demonstration is a longitudinal dance that engages many students in the class. Each student stands as a model for a vertical layer of air in an imaginary horizontal pipe. Figure 1 illustrates the arrangement with eleven students, where each student is labeled with a number from 1 to 11. They hold hands since the layers of air interact with each other in the formation of acoustic wave patterns. Alternating by gender helps to assign displacement nodes, which are specific individual students who do not move during the dance. A twelfth student is selected to be the conductor in order to set the tempo and guide the dancers. The dance is demonstrated in the accompanying video abstract. [1]

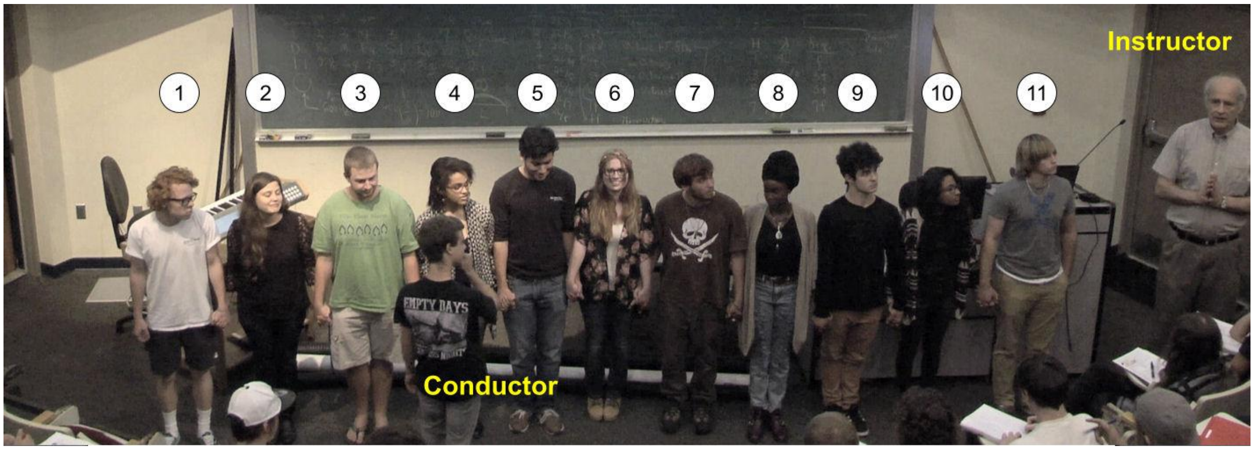

Figure 1. Students arranged in a line for dancing longitudinal waves in an open pipe. Alternating students by gender enables easy assignment of displacement nodes, e.g. the male students 3 and 9 for the second harmonic of an open pipe.

The physics

A simplest longitudinal standing wave (stationary wave) in an open pipe can be represented by a slinky where the ends alternately move inward and outward. A more sophisticated explanation for a longitudinal standing wave is that longitudinal waves reflect off each end of the pipe and the standing wave is a superposition of two longitudinal waves traveling in opposite directions. Reflections occur whenever there is a boundary between two acoustical environments. In this case, the two environments are the region inside the pipe and the free outside region. The resistance to the traveling wave, which is called the acoustic impedance, changes at boundaries. Therefore, reflections occur at both open and closed ends of pipes.

While it is easy to understand reflection at a closed end, analogous to bouncing off a wall, students often find reflections at open ends mysterious. A nice description of the open-end phenomena is provided in the *AS and A Level Delivery Guide H156/H556, Physics A Theme: Waves (August 2015)* [2]:

"When a sound wave passes through a medium and meets a boundary, the energy of the wave splits up – some of it is transmitted into the next material and some of it is reflected back (this links to the material in the medical physics part of the specification, being the way that ultrascans work). The amount of energy in the reflected wave is determined by the *acoustic impedance* of the two mediums. The point here is that the acoustic impedance of air in an open space is different to the acoustic impedance of trapped air (i.e. the air in the pipe). This difference in acoustic impedance is the reason why energy is reflected back down the tube from the open end."

Schematics for three longitudinal standing waves are shown in figure 2. The first case at the top is the fundamental (first harmonic) for an open pipe, with wavelength

$$\lambda_1^{open} = 2L\,, \tag{1}$$

where L is the length of the pipe. For this standing wave there is a single displacement node in the center of the pipe, meaning that the layer of air at the center is stationary in our model. On the other hand, there are displacement antinodes at each end, i.e. locations of maximum movement. Note that the ends move inward (blue arrows) and then outward (red arrows) opposite each other. When the ends move in, there is a compression at the center of the pipe; when the ends move outward, a rarefaction results at the pipe's center. The pressure undergoes extreme compression and rarefaction at the middle location. Therefore, the middle position, which is a displacement node, is also a pressure antinode. At the ends, where there are displacement antinodes (maximum movement), the pressure is constant at atmospheric pressure. Therefore, the ends are also pressure nodes.

Displacement and pressure nodes, with their corresponding antinodes, can be quite confusing to students. The dance helps students visualize these. If a student in the dance does not move, then that student is a displacement node. But that stationary student will get squeezed (compression) and stretched (rarefaction) making that student also a pressure antinode. Node simply means that the relevant characteristic does not change. If a student does not move, then that student is a displacement (motion) node. If a student does not get squeezed or stretched, e.g. a free student at an open end, then that student is a pressure node. The displacement nodes in figure 2 are indicated by vertical lines, while the displacement antinodes are shown by the horizontal lines with the blue and red arrows. A displacement node is a pressure antinode, and a displacement antinode is a pressure node. They are opposites of each other. When there is more than one pressure node, the colored arrows in figure 2 indicate the simultaneous motions for the different regions.

The middle wave in figure 2 is the second harmonic (first overtone) for an open pipe, with wavelength

$$\lambda_2^{open} = L. \tag{2}$$

In general, the nth mode for an open pipe has wavelength $\lambda_n^{open} = \frac{2L}{n}$. The third case (bottom wave) is the fundamental for a closed pipe, where the wavelength is

$$\lambda_1^{closed} = 4L. \tag{3}$$

The general formula for a closed pipe is $\lambda_m^{closed} = \frac{4L}{m}$, where m is odd. The relative frequencies for the three standing waves in figure 2 are as follows: f for the top wave, $2f$ for the middle wave, and $\frac{f}{2}$ for the bottom wave. These three relative frequencies are inversely proportional to the wavelengths found in equations (1), (2), and (3).

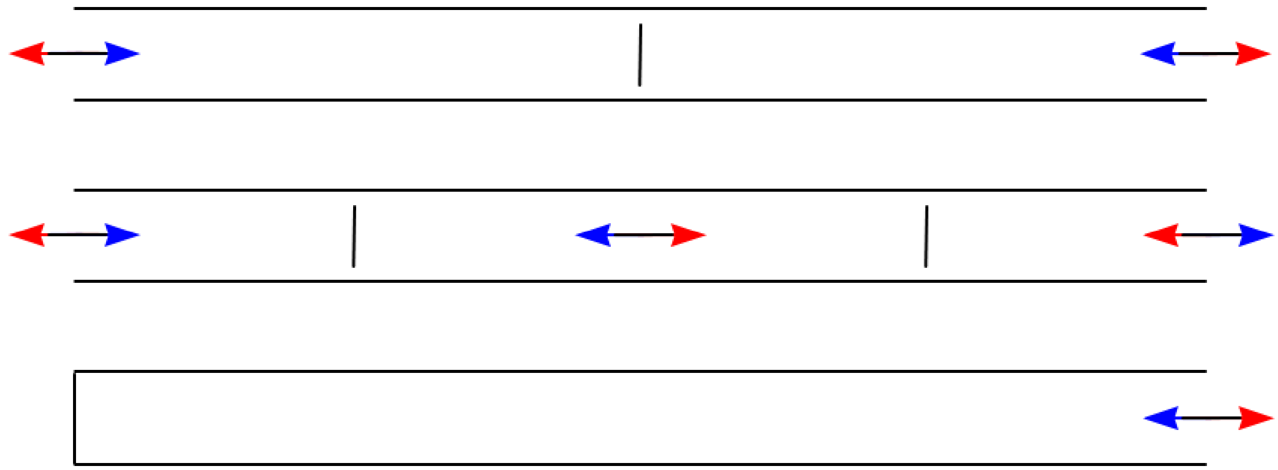

Figure 2. Schematic for the fundamental mode of an open pipe (top), the second harmonic mode of vibration for an open pipe (middle), and the fundamental mode for a closed pipe (bottom).

The choreography

The choreography for the three dances are shown in figures 3a, 3b, and 3c. Figure 3a illustrates the fundamental for an open pipe. The student in the middle, student 6, is asked to remain stationary while the students on either side move inward to squeeze the center student, then move outward to stretch the middle student. Students in the audience find this fun dance quite amusing as the middle person gets squeezed and stretched.

Though it has been mentioned that each dance has a relative frequency relationship, the more important feature of the dance is the visualization of the motion and identification of nodes/antinodes. We do not worry about relative frequencies in our video. [1]

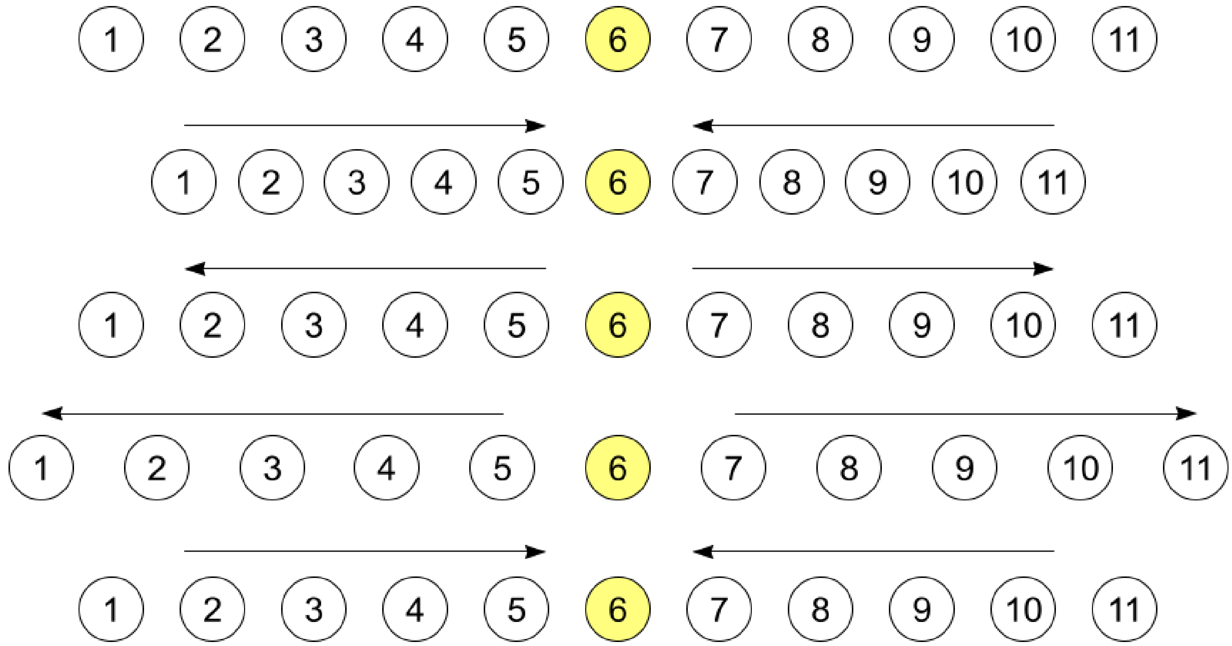

Figure 3a. The choreography for dancing the fundamental in an open pipe. Students at each end first move inward to squeeze student 6 at the center and then move outward to stretch student 6.

Figure 3b reveals the choreography for the second harmonic of an open pipe. Now there are two displacement nodes, one at position 3 and the other at position 9. Note that these displacement nodes are located at approximate positions due to our small number of students. The distance between 3 and 9 (a half wave) should be more precisely twice the distance from 1 to 3 (a quarter wave) or 9 to 11. But the approximation is close enough to visualize all the important physics.

For this mode, when 3 gets stretched, 9 gets squeezed and vice versa. This dance can be confusing due to the more complicated motion. There are three student subgroups that move: students 1-2, students 4-to-8, and students 10-11. Students 3 and 9 do not move. The conductor's hands should move outward and inward, left and right hands moving opposite each other. The students in each of the three different sections should focus their attention on the appropriate conductor's hand. Then, it is easier for the students to dance the wave.

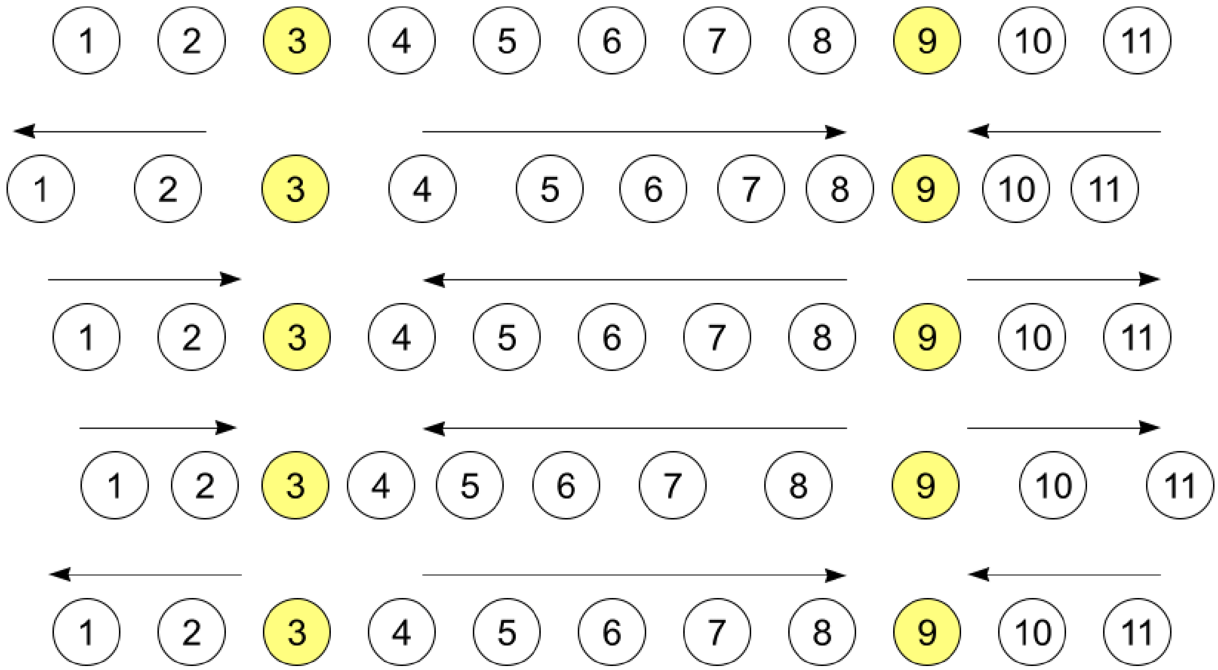

Figure 3b. The choreography for dancing the second harmonic in an open pipe. Now there are two displacement nodes, 3 and 9. When 3 gets stretched, 9 gets squeezed and vice versa.

Finally, figure 3c depicts the fundamental for a closed pipe. Student 1 should be up against a wall, attached to the wall so to speak. Our class has a railing near the wall so student 1 grabs on to this railing and does not move. This dance is actually the easiest as all the other students 2-to-11 move in to squeeze the poor person at the wall and then move out to stretch the wall person. Again, students find the squeezing and stretching of a student entertaining to watch. This last example has a frequency equal to half that of the fundamental for the open pipe of the same length. However, as mentioned earlier, in our video [1] we do not worry about frequencies and instead stress the visualization of the movement and displacement nodes.

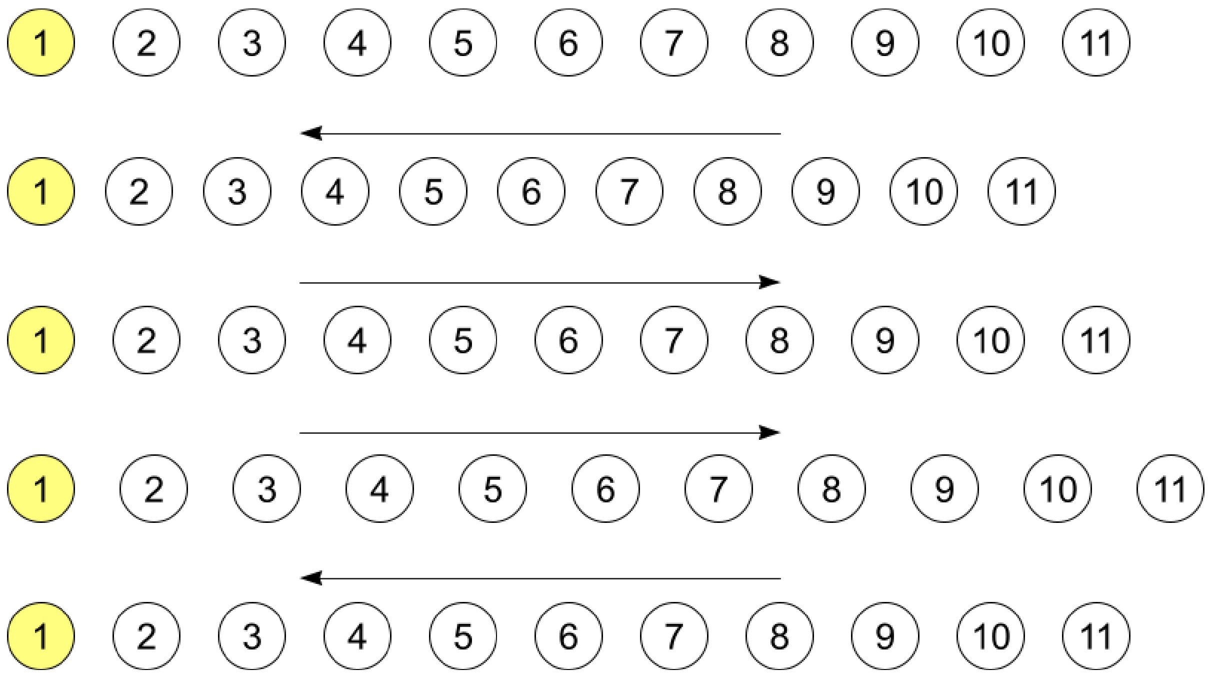

Figure 3c. The choreography for dancing the fundamental a closed pipe. Student 1 is a displacement node at a wall in the classroom. The other students first move in to squeeze student 1 and then move out to stretch student 1.

Incorporating end effects at open ends

Teachers may wish to incorporate open-end effects in the activity. It has been known for over 100 years that the pressure nodes at the ends of open pipes are located a little beyond the physical ends of the pipe. In 1896 Lord Rayleigh [3] gave the correction for each open end as $\Delta L = 0.6r$ where r is the radius of the pipe and the wavelength $\lambda \gg r$. This correction translates to using an effective pipe length of $L' = L + 2(0.6r) = L + 1.2r$ for a pipe with two open ends, where L is the physical length of the pipe. A half century later, Levine and Schwinger [4] used considerable amounts of mathematical physics to study end effects in detail, perhaps being the first to recognize that end corrections depend on frequency. [5]

In the limit where $kr \equiv \dfrac{2\pi r}{\lambda} << 1$, Levine and Schwinger show [5] that the end correction for a pipe open on each end is

$$L' = L + 2(0.6133) = L + 1.2266r\,, \tag{4}$$

which is surprisingly close to Rayleigh's estimate over 50 years earlier. For a typical classroom tube where the physical length is $L = 100\,\text{cm}$ and radius $r = 2\,\text{cm}$, the fundamental wavelength $\lambda = 2L' = 2(L + 1.2r) = 2(101.2) = 202.4cm$, and

$$kr \equiv \frac{2\pi r}{\lambda} = \frac{2\pi r}{2L'} = \frac{\pi r}{L'} = \frac{3.1416(2\,\text{cm})}{101.2\,\text{cm}} = 0.06 << 1\,. \tag{5}$$

Equation (5) indicates that using $L' = L + 1.23r$ from equation (4) is appropriate for typical pipes encountered in class, however, authors often round off to $0.61r$ for a single open end and $1.22r$ for two open ends.

To incorporate end effects in the dancing activity, place a mark on the floor to indicate the physical ends of the pipe. Then, a little extra distance away from the physical ends, mark the effective ends of the pipe. The students can then dance the fundamental for an open pipe with the end pressure nodes being a little farther out in order to illustrate the end effect. For a 1-meter open pipe with a 2-centimeter radius, the end correction is

$$100\% \times \left[\frac{L' - L}{L}\right] = 100\% \times \frac{1.2r}{L} = 100\% \times \frac{1.2(2\ \text{cm})}{100\ \text{cm}} = 2.4\%\,. \tag{6}$$

Concluding remarks

The dances enable students to visualize longitudinal waves in both open and closed pipes. Dancing just the three cases described in this paper is enough to get the basic physics across. A more ambitious project would be to expand the number of students and work out the choreography for higher harmonics. The spacing rule is that the distance from a node to an antinode is a quarter wave and the distance from a node to a node or an antinode to an antinode is a half wave.

Students might be reluctant at first to come up to the front of the class. But the fact that many students are needed helps greatly. Holding hands is a pleasant surprise that they generally like. Watching specific students get squeezed and stretched is most entertaining. Enjoy the accompanying video abstract [1] to see this educational fun activity in action.

Acknowledgments

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Reference

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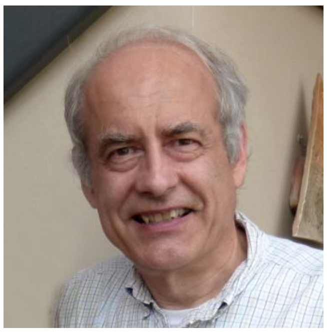

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