A Reliable Wave Convention for Oppositely Traveling Waves

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A Reliable Wave Convention for Oppositely Traveling Waves

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Care must be taken representing waves traveling in opposite directions. Erroneous results can occur when combining waves with the popular convention where a wave traveling to the right is $\Psi_R(x,t) = A \sin(kx - \omega t)$ and a left-traveling wave is $\Psi_L(x,t) = A \sin(kx + \omega t)$. An easily overlooked pitfall is pointed out and a way to avoid it with a secure convention is presented. As an exercise in critical thinking, students can be presented with the pitfall and asked to find a way out.

Background

A common way to introduce a traveling disturbance is to first shift a function $f(x)$ along the positive x-axis a distance d by replacing $f(x)$ with $g(x) = f(x - d)$. Students can easily appreciate this effect if you sketch a peaked function $f(x)$ at the origin so that $f(0)=$ peak. Refer to Fig. 1(a). Then, from $g(x) = f(x - d)$ we find $g(d) = f(d - d) = f(0) =$ peak, indicating that the peak has shifted a distance d along the positive x-axis. Finally, setting $d = vt$ gives the right-traveling disturbance $f(x - vt)$. Switching the sign in front of the speed leads to the left-traveling wave $f(x + vt)$. See Fig. 1(b) where the pulse at the origin (left) now moves to the right with speed v and the shifted pulse (at right) moves to the left with the same speed v.

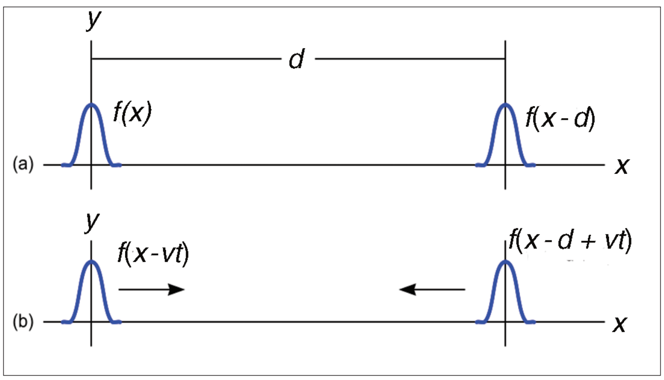

Fig. 1. (a) Shifting a pulse function to the right a distance *d*. (b) Including *vt* with opposite signs such that the pulse at the origin moves to the right and the shifted pulse moves to the left.

Introductory textbooks[1-8] often present students with sine waves using the convention described above. Sine waves traveling to the right and left, respectively, are given by

$$y_R(x,t) = A \sin(kx - \omega t), \quad (1a)$$

$$y_L(x,t) = A \sin(kx + \omega t), \quad (1b)$$

where $kx - \omega t = k(x - vt)$. The familiar parameters k and ω are the wave number and angular velocity, respectively. The common description $2A \sin(kx)\cos(\omega t)$ related to standing waves can be obtained by adding Eqs. (1a) and (1b). However, the example in the following section demonstrates an application leading to an unsatisfactory result.

A pitfall

To illustrate the treacherous nature of the convention described above for waves traveling to the right and left, first see Fig. 2 for an arrangement where two speakers are separated by a distance $2d$. The speakers emit sine waves in phase, which are picked up by the microphone at the middle position between the two speakers.[9]

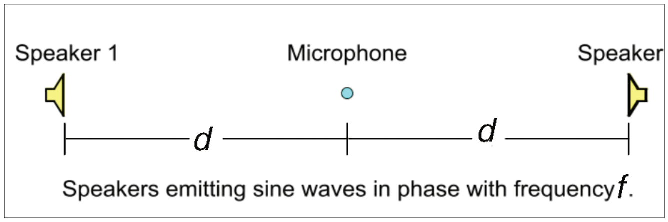

Fig. 2. Two speakers separated by a distance 2*d* with a microphone midway between them. The speakers emit sine waves in phase with frequency *f*, which are picked up by the microphone.

Let the wave leaving the left speaker be $y_R(x,t) = A \sin(kx - \omega t)$, where the subscript R designates that the wave travels to the right. For the wave emitted from the right speaker, we arrive at the function by shifting a sine wave a distance $2d$ to the right and changing $-\omega t$ to $+\omega t$ so that the wave moves to the left: $y_L(x,t) = A \sin[k(x - 2d) + \omega t]$. The superposition of the waves at the microphone, i.e., at $x = d$, is then

$$y(d,t) = A \sin(kd - \omega t) + A \sin(-kd + \omega t) = 0, \quad (2)$$

which disagrees with the experimental observation. The waves at the microphone undergo constructive interference since the sources are emitting in phase and the waves are traveling the same distance in order to reach the center. Equation (2) indicates an incorrect result since it predicts destructive interference at the center point.

An inspection of Figs. 3(a) and (b) reveals that the convention does not work for the sine wave, which is an odd function. A disguised phase shift has been introduced, as illustrated in Fig. 3(b), where a leading trough travels from left to right, while a leading crest travels from right to left. However, the correct result is obtained for Fig. 3(a), where the cosine function (an even function) is used. In the next section we discuss Figs. 3(c) and (d).

A correct approach

Jonathan A. Jones at the University of Oxford points out to his students that there are several wave conventions: the good, the bad, and the ugly.[10] We have employed in the previous section a convention that Jones lists as bad.[10] He also reminds us that the rules presented in Eqs. (1a) and (1b) are frequently used and closely linked to d'Alembert's solution[11] $y(x,t) = f(x - vt) + g(x + vt)$ of the wave equation in one spatial dimension.

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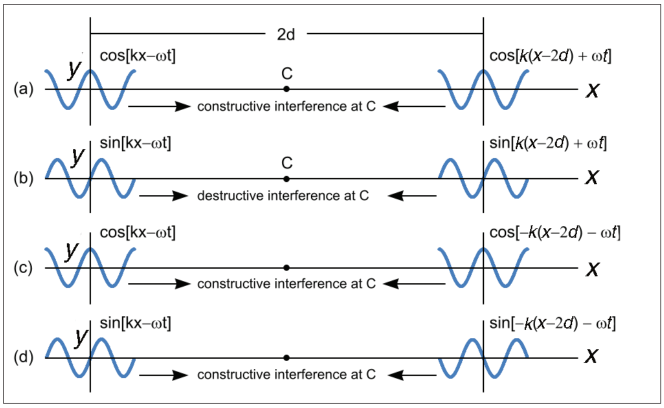

Fig. 3. Cosine and sine waves are first shifted a distance 2*d* to the right, cosine waves for (a) and (c); sine waves for (b) and (d). The convention where the sign of ω changes is used for the left-traveling waves in (a) and (b); the convention where the sign of *k* changes is employed for the left-traveling waves in (c) and (d). The latter convention gives the correct conclusion that constructive interference occurs at the center point for both the cosine (c) and sine (d) waves.

The good convention given in Ref. 10 is the one used in advanced physics where the sign change for the left-traveling wave is made by reversing the sign of k rather than the sign of ω. We will refer to reversing the sign of k as the wave-vector convention since in three dimensions one finds $\pm\ \mathbf{k}\cdot\mathbf{r} - \omega t$. The good convention is readily found in current texts for upper-level courses in optics,[12] electromagnetic theory,[13-14] and quantum mechanics.[15-16] With the wave-vector convention, Eqs. (1a) and (1b) become

$$y_R(x,t) = A \sin(kx - \omega t), \tag{3a}$$

$$y_L(x,t) = A \sin(-kx - \omega t). \tag{3b}$$

Equation (2) with $x = d$ then becomes

$$y(d,t) = A \sin(kd - \omega t) + A \sin(+kd - \omega t) = 2A \sin(kd - \omega t), \tag{4}$$

representing constructive interference, in agreement with the experiment. Refer to Figs. 3(c) and (d) for an illustration of the correct result for both cosine and sine waves.

Conclusion

Conventions that switch the sign of ωt to represent waves traveling to the left introduce an inadvertent hidden phase shift, which can lead to misconceptions. We have shown that the common wave convention for traveling waves found frequently in introductory texts can result in the erroneous conclusion given by Eq. (2). A challenge can be posed to students to properly describe the physics with the common convention by introducing a phase shift. When the waves leave in phase, the waves interfere constructively at the microphone, which is located at the midpoint between the two speakers. However, the wave convention used with the odd sine functions to arrive at Eq. (2) introduces an accidental phase shift of 180° for one of the waves.

Using the wave-vector convention, one can proceed in a straightforward fashion to shift the source of the wave and reverse the traveling direction to arrive at the correct result. The analysis of wave conventions can sharpen a student's critical thinking, forcing the student to ponder deeply in order to understand the physics behind each wave scheme. Finally, students will appreciate the wave-vector convention, which is the more reliable approach for analyzing problems with waves traveling in opposite directions.

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