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Video of scenery during a total eclipse: luminance and effects of solar limb darkening

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Abstract

A video of darkening scenery during the 15 minutes prior to totality of the 21 August 2017 total solar eclipse is presented and discussed. The luminances of the scenes at one-minute intervals are obtained by averaging pixel data of stills from the video. The experimental results are compared to the theoretical model of uniform radiance from the exposed area of the Sun during the eclipse. The measured values deviate from the model since light from the solar disk decreases towards the edges of the Sun.

The darkening world of a total solar eclipse

The total solar eclipse is one of nature's most spectacular shows in the sky. Mark Twain includes a climactic scene in his novel A *Connecticut Yankee in King Arthur's Court*. In the story a 19th-century Yankee from Hartford, Connecticut suddenly finds himself in England at the time of King Arthur. Knowing about an eclipse, he escapes being burned at the stake by threatening the court [1].

> 'Go back and tell the King that at that hour I will smother the whole world in the dead blackness of midnight; I will blot out the sun, and he shall never shine again; the fruits of the earth shall rot for lack of light and warmth, and the peoples of the earth shall famish and die, to the last man!'

Historically, there was no actual eclipse on 21 June, 528 as stated in the novel. Three total eclipses over England in the 5th and 6th centuries occurred [2] respectively in the years 413, 458, and 594.

To capture the awe of the darkening world described by Mark Twain, I shot a video [3] of the local scenery for the 15 minutes just before the onset of totality during the 21 August 2017 total solar eclipse. I set my digital video camera on a tripod and selected manual mode so that the camera would not automatically adjust for the decreasing available light. My pilot friend Bruce Greene flew

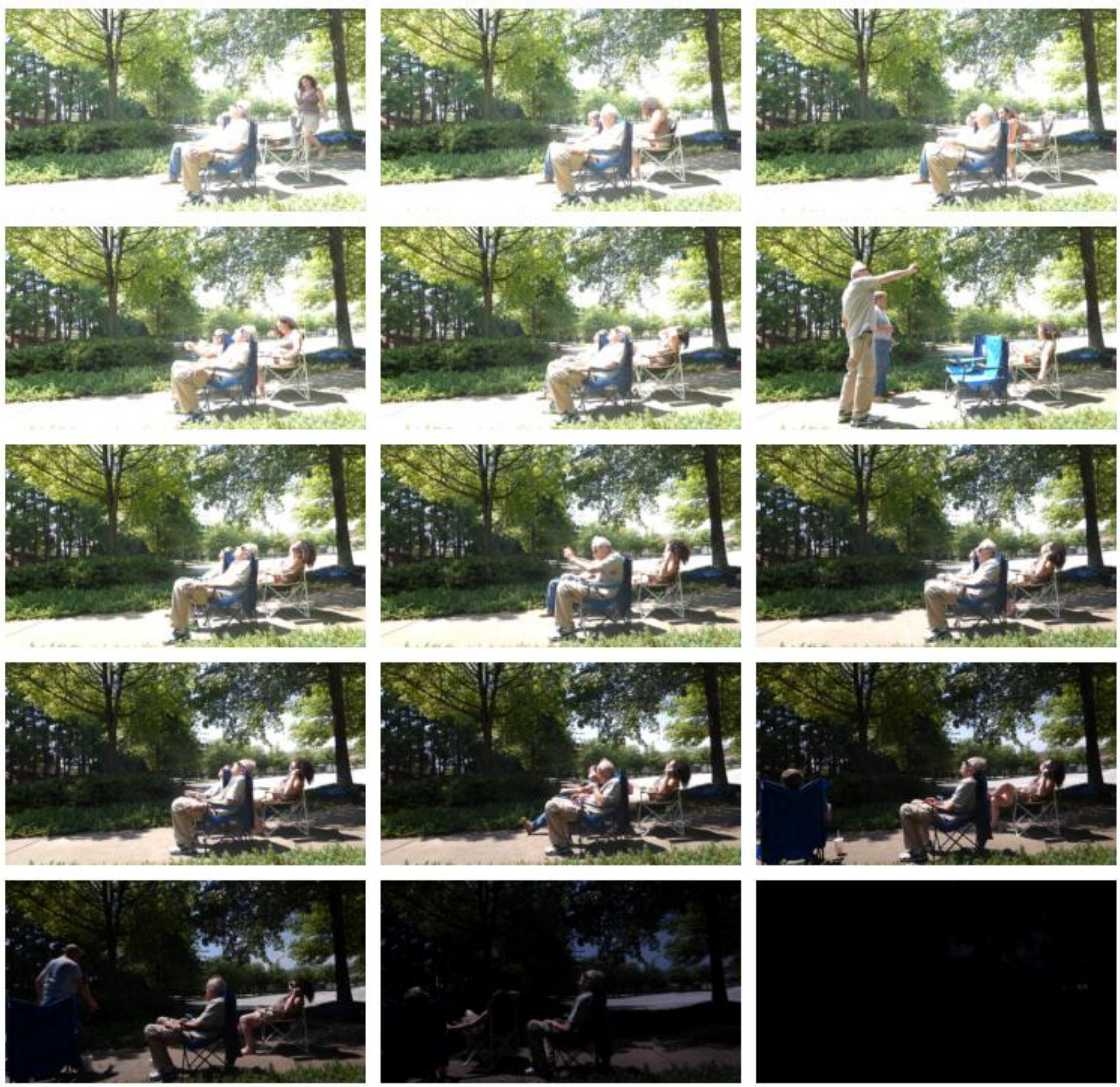

Figure 1. A series of stills from a video of the surroundings during a total solar eclipse. The time between each still is 1 minute. The last image is at the onset of totality. The author is closest to the camera. A slide show of these photos at high resolution is included in Reference 3.

his girlfriend April and me ninety miles from our hometown in Asheville, North Carolina, USA to

Greenville, South Carolina, to be in the path of totality. See figure 1 for a series of 15 stills from the video spaced one minute apart, arranged from right to left and in rows. Since the video frame rate was 30 frames per second, each still has an exposure of 1/30 second. Be sure to watch the exciting video from which the stills came and view the included high resolution image gallery [3]. The video includes a sped-up version as totality approaches.

For one with no understanding of an eclipse, the series of images in figure 1 would be quite alarming. Temperature drops accompany the darkening surroundings and animals get confused as to the time of day. In the Mark Twain story, King Arthur readily orders the Yankee to be released and to name the terms in order to bring the sunlight back. The negotiations need to take place in minutes since the duration of totality is typically 2 or 3 minutes with the longest possible eclipsed Sun being 7.5 minutes [4]. In the next section luminance measurements are obtained from the stills and plotted as a function of time.

Radiance, luminance, and brightness

Radiance refers to the total amount of electromagnetic radiation with units Watts per square meter per steradian. Luminance is the weighted electromagnetic radiation in units lumens per square meter per steradian, based on the sensitivity of visual receptors in the human eye [5]. The brightness is the nonlinear perception of luminance due to processing in the eye and brain. See figure 2, based on a slide by Gordon Kindlmann [6].

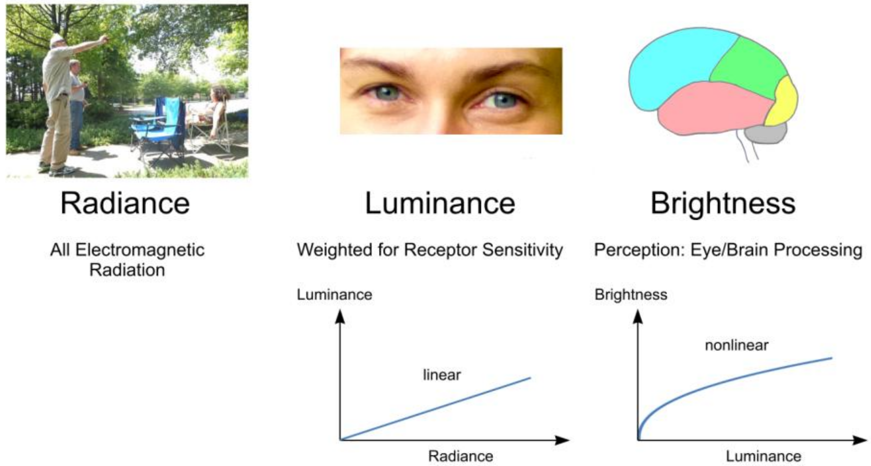

Figure 2. The relationships among radiance, luminance, and brightness adapted from a figure by Gordon Kindlmann [6].

The units are not important since ratios will be taken throughout this paper. An important point is that the luminance is a linear function of the radiance. So if one has two identical sources, the luminance doubles as well as the radiance. However brightness, the perception of luminance, is a nonlinear function [7] of the luminance due to image processing by the eye and brain. The eye receptors adjust sensitivity depending on available light through "a process called dark adaptation, which causes the eye to increase its sensitivity in the dark" [8]. The brain completes the processing of visual information from the eye. Digital camera sensors respond to radiance. However, in the processing to form a jpg or png image suitable for viewing on a computer monitor, the raw data is transformed to approximate the perception of brightness by the human eye. The next section will discuss how to extract luminance data from the transformed pixel values in an image taken by a digital camera.

Measuring luminance from pixels

As pointed out in the previous section, humans perceive brightness in a nonlinear fashion. When the human eye views a scene illuminated by one light source, two identical light sources, three, etc., the perception is not twice as bright, three times as bright, etc. The processing by the eye and brain leads to a perception of brightness which represents a nonlinear compression of luminance. Students will be more familiar with a similar compression involving sound, where acoustic intensity ranging from $10^{-12}\ \mathrm{Wm}^{-2}$ (threshold of hearing) to $10^{-2}\ \mathrm{Wm}^{-2}$ ('disco/rock gig') [9] is mapped to a perceived loudness span from 0 to 100 decibels (dB).

The image processing in a digital camera mirrors human perception of brightness with colour stored in the form of red (R), green (G), and blue (B) pixel values, each ranging from 0 to 255. Microsoft and Hewlett Packard (HP) in 1996 "recognised the requirement for defining a common colour space for use in the embryonic industry" of digital cameras [10]. The white point and colour "as described in Recommendation ITU-R BT.709 (Rec 709), was adopted" and "this colour space was defined as *the Standard* RGB colour space or sRGB" [10]. The procedure for obtaining the luminance from pixel values of this standard RGB colour space consists of the three steps outlined below.

Step 1. Finding the average sRGB for the scene

First, the average sRGB is found for the still image. The original images in figure 1 have dimensions 1920 x 1080 pixels. A 480 x 330 pixel region in the upper left corner was cropped out for the measurements since that portion of the scene was constant throughout, while in other portions people moved around. This cropped section appears in figure 3.

Figure 3. Cropped image for luminance analysis during the total solar eclipse.

A cropped image of direct sunlight on the cement foreground was not possible due to the saturation of the R, G and B pixels at their maximum values of 255 for many of the images during the solar darkening. Therefore, a region of diffuse reflection was chosen. The assumption is made that the luminance from the diffuse reflected light is proportional to the luminance of the direct solar light. The cropped image can be analyzed by a commercial program such as MatLab to determine the average sRGB values for the entire scene. However, a free online program by Matthias Klein (http://matkl.github.io/average-color/) gives the same results. The online program has the advantage that images can easily be dragged and dropped into a box to very quickly find the average sRGB values [11] with no additional computer code as required by MatLab. The results are listed in Table 1 as sR, sG, and sB.

Table 1. Pixel averages sRGB, normalized values $sRGB_{norm}$, linearised RGB_{lin}, and relative luminance. The calculations are explained in the text.

Time (min)	sR	sG	sB	sR_{norm}	sG_{norm}	sB_{norm}	R_{lin}	G_{lin}	B_{lin}	$L_{relative}$
75	176	187	132	0.690	0.733	0.518	0.434	0.497	0.231	1.00
76	170	181	124	0.667	0.710	0.486	0.402	0.462	0.202	0.93
77	164	175	117	0.643	0.686	0.459	0.371	0.429	0.178	0.86
78	158	168	109	0.620	0.659	0.427	0.342	0.392	0.153	0.78
79	151	161	101	0.592	0.631	0.396	0.309	0.356	0.130	0.71
80	142	152	91	0.557	0.596	0.357	0.270	0.314	0.105	0.62
81	134	144	81	0.525	0.565	0.318	0.238	0.279	0.082	0.55
82	121	130	72	0.475	0.510	0.282	0.191	0.223	0.065	0.44
83	110	118	66	0.431	0.463	0.259	0.156	0.181	0.054	0.36
84	97	104	57	0.380	0.408	0.224	0.120	0.138	0.041	0.27
85	83	89	47	0.325	0.349	0.184	0.087	0.100	0.028	0.20
86	67	72	36	0.263	0.282	0.141	0.056	0.065	0.018	0.13
87	50	53	25	0.196	0.208	0.098	0.032	0.036	0.008	0.07
88	29	31	14	0.114	0.122	0.055	0.012	0.014	0.004	0.03
89	9	10	6	0.035	0.039	0.024	0.003	0.003	0.002	0.01
90	0	0	0	0.000	0.000	0.000	0.000	0.000	0.000	0.00
91	0	0	0	0.000	0.000	0.000	0.000	0.000	0.000	0.00
92	0	0	0	0.000	0.000	0.000	0.000	0.000	0.000	0.00
93	12	11	6	0.047	0.043	0.024	0.004	0.003	0.002	0.01
94	32	34	17	0.125	0.133	0.067	0.014	0.016	0.006	0.03
95	52	56	31	0.204	0.220	0.122	0.034	0.040	0.014	0.08
96	70	76	45	0.275	0.298	0.176	0.061	0.072	0.026	0.14
97	86	93	55	0.337	0.365	0.216	0.093	0.109	0.038	0.22
98	101	108	66	0.396	0.424	0.259	0.130	0.150	0.054	0.30
99	114	122	77	0.447	0.478	0.302	0.168	0.195	0.074	0.39

Step 2. Linearising the normalized sRGB values

The next step is to normalize each sRGB value by dividing by 255, since pixels values range from 0 to 255. Then, the normalized sRGB values need to be linearised. The recommended procedure for linearising the normalized values sRGB is given by [12]

$$f(x) = \begin{cases} \dfrac{x}{12.92} & 0 \le x \le 0.04045 \\ \left[\dfrac{x+0.055}{1.055}\right]^{2.4} & 0.04045 < x \le 1 \end{cases} \tag{1}$$

where x is sR_{norm}, sG_{norm}, or sB_{norm}, and the corresponding $f(x)$ values are the linearised R_{lin}, G_{lin}, and B_{lin} entries in Table 1.

Step 3. Obtaining the luminance from the linearised RGB values

To obtain the luminance from the linearised values R_{lin}, G_{lin}, and B_{lin}, the following prescription is used [12].

$$L = 0.2126R_{lin} + 0.7152G_{lin} + 0.0722B_{lin} \tag{2}$$

The largest coefficient is for the green since the human eye is most sensitive in the green region of the visible spectrum. Red comes next in sensitivity and blue last. Think of equation (1) as uncompressing the nonlinear sRGB values and equation (2) as applying the necessary weighting factors to arrive at the luminance. The last column in Table 1 lists the luminances relative to the luminance at time 75 minutes into the eclipse, a time 15 minutes prior to the onset of totality.

A plot of the relative luminance as a function of time is provided in figure 4. Totality begins at 90 minutes and lasts for the duration of about 2 minutes. At the end of totality, the luminance begins to increase as more and more light emerges from the Sun. Note the nice symmetry about the middle of the eclipse. In the next section, the empirical data will be compared to a simple model based on the area of the unexposed Sun.

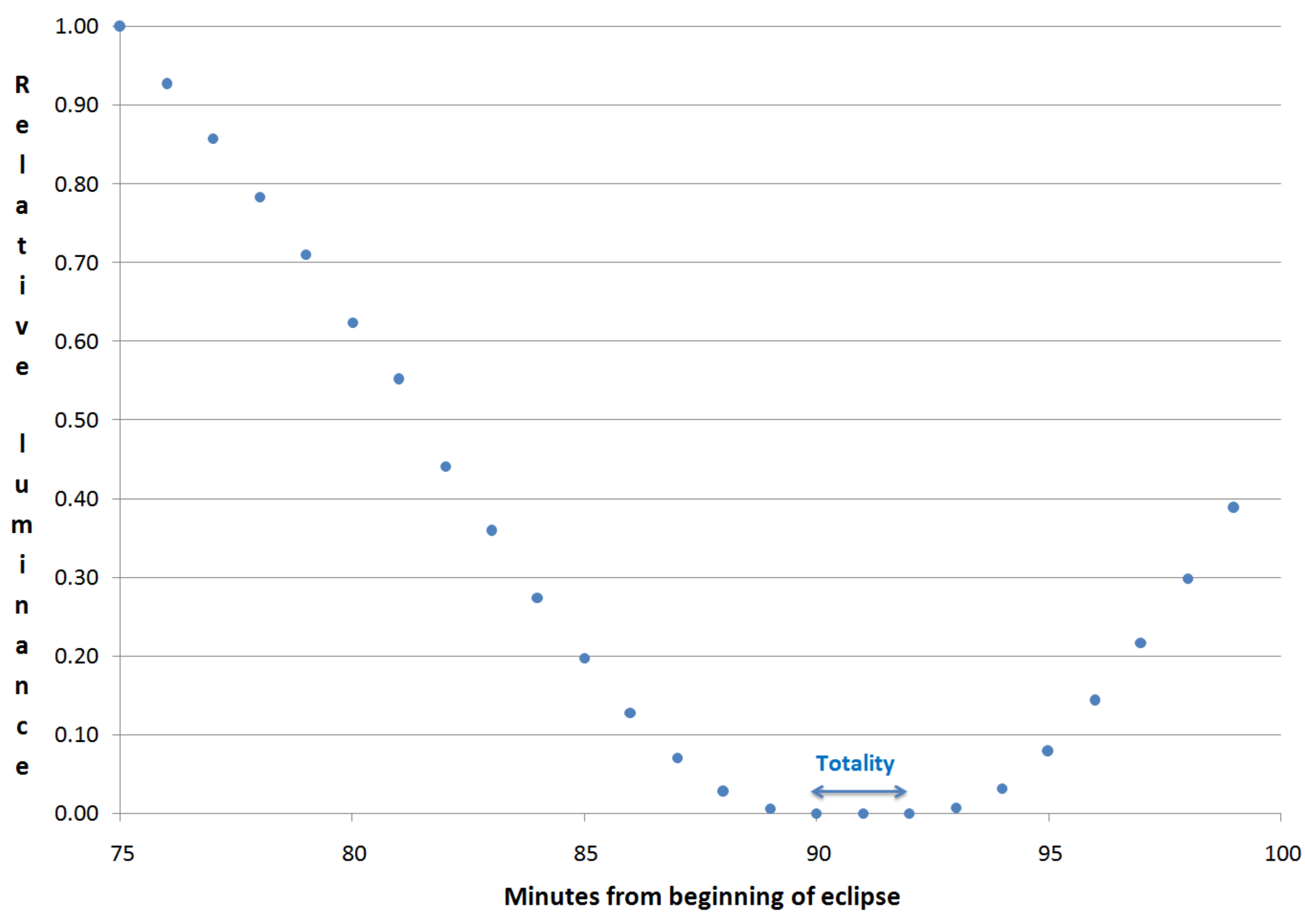

Figure 4. A plot of the relative luminance of a scene during a total eclipse where the reference luminance is at 75 minutes, a time 15 minutes prior to the onset of totality.

Simplest theoretical model

In the simplest theoretical model the luminance is calculated by determining the area of the Sun that is unexposed. This model neglects variations of luminance due to sunspots, prominences, and solar limb darkening. Solar limb darkening refers to the fact the light coming from the Sun "decreases from the centre of the disc toward the limb" [13]. In the simple model of a uniform solar disc, if the unexposed area of the Sun doubles, the radiance doubles as well as the luminance.

The model and associated calculations presented here represent a simplified version of a publication by Hughes in the *Journal of the British Astronomical Association* [14]. A similar treatment appears in a paper by Möllmann and Vollmer in the *European Journal of Physics* [15]. In our model presented below, the Sun and Moon are taken to have the same apparent size in the sky and the mathematics is simpler, avoiding the use of the law of cosines. Therefore, the model presented below is more easily accessible for introductory students. However, it is important to point out that the simple model "corresponds to a duration of totality of zero" [15], when the moon precisely covers the Sun for an instant. The actual relative apparent size of the Moon is slightly larger than 1. See Möllmann and Vollmer [14] for all general cases of apparent lunar sizes that describe all types of solar eclipses: partial, annular, and total.

Our model appears in figure 5, where the discs of the Sun and Moon as they appear in the sky are taken to be the same size, each with radius 1. The goal is to calculate the area of the exposed Sun that forms the crescent ABGCA. Students will be surprised that this area is not hard to calculate using elementary geometry and trigonometry.

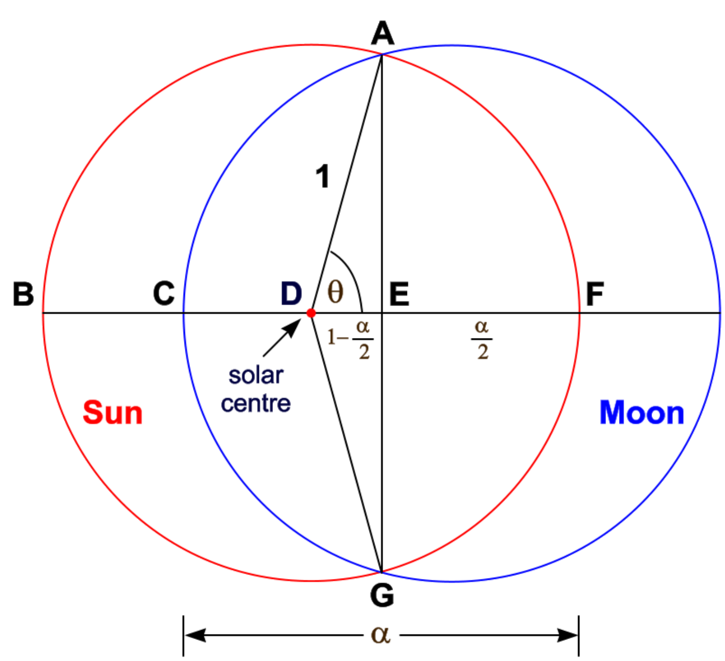

Figure 5. The Moon covering the Sun during an eclipse where the covered solar disc is the region ACGFA. The exposed Sun is the region ABGCA. The parameter α ranges from 0 to 2 from the time the Moon just begins to cover the Sun to the time the Moon completely covers the Sun.

The plan is to first find the area ACGFA of the covered Sun. Then, subtract this area from the area $\pi \cdot 1^2$ of the Sun in our units where the radius of the Sun is 1. The area ACGFA of the covered Sun is twice the area of the chord section AEGFA. This chord section is equal to the angular pie section ADGFA minus the triangular region ADGEA. The area of the angular pie section ADGFA is

$$A_{ADGFA} = \pi 1^2 \frac{2\theta}{2\pi} = \theta , \tag{3}$$

where θ is measured in radians. The area of the triangular section ADGEA is two times the area of the triangle ADEA.

$$A_{ADGEA} = 2 \cdot \frac{1}{2} \cos\theta \sin\theta = \cos\theta \sin\theta \tag{4}$$

As noted above, the chord section AEGFA = ADGFA - ADGEA,

$$A_{AEGFA} = A_{ADGFA} - A_{ADGEA} = \theta - \cos\theta \sin\theta . \tag{5}$$

The area of the covered Sun is twice the area given in equation (5).

$$A_{\text{covered}} = A_{ACGFA} = 2A_{AEGFA} = 2\theta - 2\cos\theta \sin\theta \tag{6}$$

Finally, the area of the exposed Sun ($A_{\exp osed}$) is $\pi \cdot 1^2 - A_{\text{covered}}$,

$$A_{\exp osed} = \pi - 2\theta + 2\cos\theta \sin\theta . \tag{7}$$

A remaining challenge is to express equation (7) in terms of the parameter α which is proportional to the time as the Moon moves across the Sun. When the eclipse begins, $\alpha = 0$; when the eclipse reaches the onset of totality, $\alpha = 2$. Let $t = 0$ at the start of the eclipse. At our location, totality was reached at $t = 90 \text{ min}$. Therefore

$$\alpha = \frac{t \text{ (in min)}}{45 \text{ min}} . \tag{8}$$

The parameter α is related to the relevant trig functions as follows.

$$\cos\theta = 1 - \frac{\alpha}{2} = \frac{2-\alpha}{2} \qquad (9a)$$

$$\sin\theta = \sqrt{1-\cos^2\theta} = \frac{\sqrt{4\alpha-\alpha^2}}{2} \qquad (9b)$$

With these trig relations, equation (7) becomes

$$A_{\exp osed} = \pi - 2\cos^{-1}\left[\frac{2-\alpha}{2}\right] + \left[\frac{2-\alpha}{2}\right]\sqrt{4\alpha-\alpha^2}\ , \qquad (10)$$

where α is related to time by equation (8).

Figure 6 is plot of the relative luminance starting at 75 minutes, i.e. 15 minutes before the onset of totality. The relative luminances are referenced to the luminance at 75 min for both the experimental data and for the theoretical curve. The theoretical plot during the 15 minutes prior to totality shown in figure 6 is surprisingly linear. Equation (10) does not appear to be linear in any obvious way. However, if the derivative is taken to find the slope, the result is

$$\frac{dA_{\exp osed}}{d\alpha} = -\sqrt{4\alpha-\alpha^2}\ . \qquad (11)$$

From t = 75 min to t = 90 min, α ranges from 75/45 = 1.67 to 90/45 = 2.00. The slope given by equation (11) ranges respectively from -1.97 to -2.00, very close to being constant.

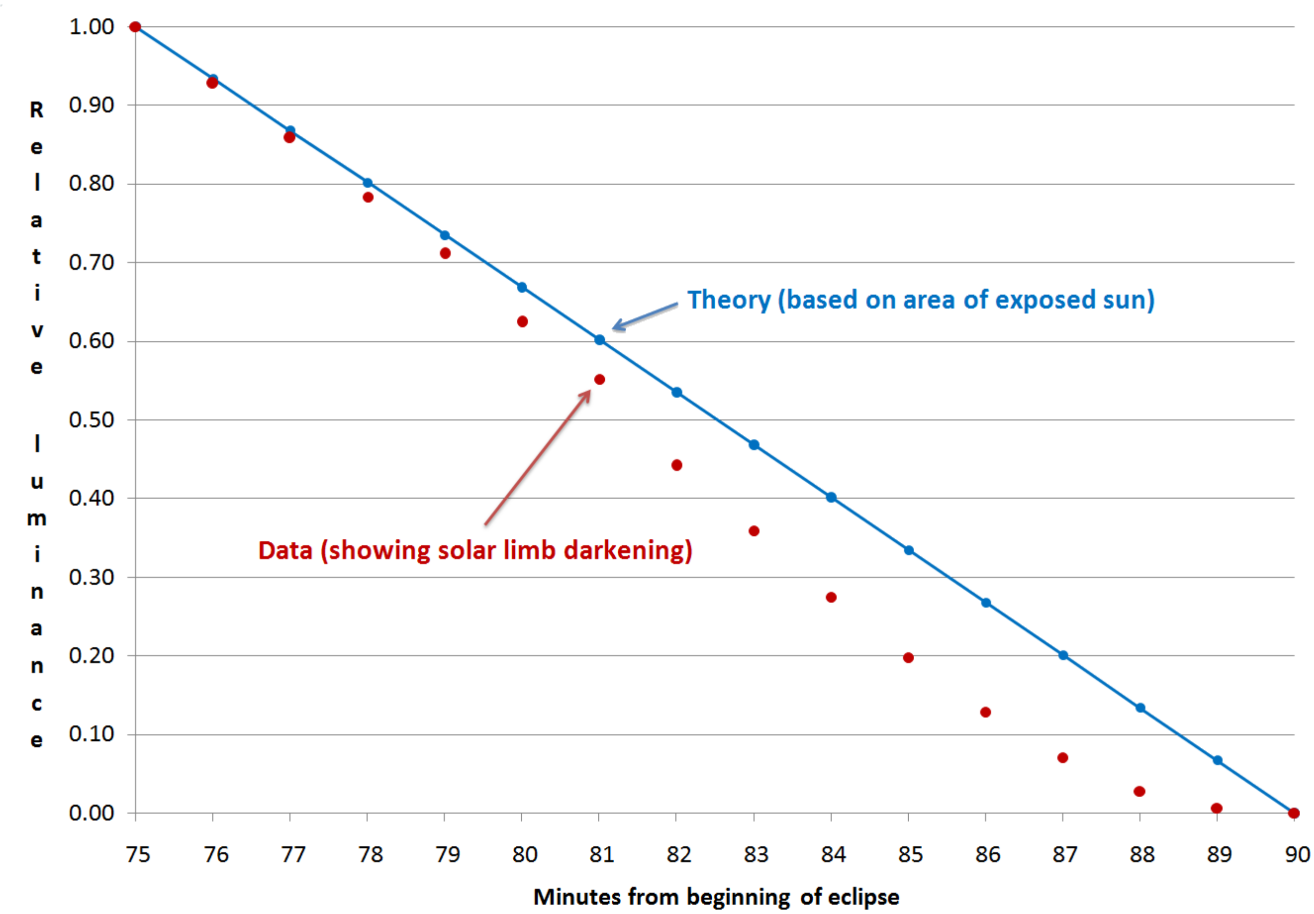

Figure 6. Theoretical (blue) and experimental (red) plots of relative luminance for the last 15 minutes before the onset of totality (90 minutes).

The measured values for the luminances drift below the theoretical curve due to solar limb darkening. i.e. the regions towards the edges (the limb) of the Sun are darker. When light is measured from the center of the solar disc (defined as 0°), the light emerges from "the hottest part of the photosphere. But when we examine the limb, the relatively darker portion of the solar disc, we receive radiation coming from the higher layers of the solar photosphere" [16]. Limb darkening is wavelength dependent. The ratio of radiation at the very oblique angle of 87° coming from the higher layers of the photosphere compared to the direct path at 0° emerging from the lower photosphere ranges from about 0.2 for 400 nm (blue extreme of the visible spectrum) to about 0.4 for 700 nm (red extreme of the visible spectrum) [17]. It is interesting to note that Sir Arthur Eddington proposed an "approximation of solar limb darkening for light integrated over all wavelengths" [18] that predicts the relative radiation dropping from the 1.00 value at 0° to 0.40 at 90°.

The limb darkening evident in figure 6 is due to integrative effects of wavelengths and crescent solar areas. A similar pair of plots is found in the paper by Möllmann and Vollmer [15] for the 29 March 2006 total solar eclipse with a path over southern Turkey. Möllmann and Vollmer measured light with a digital light meter. More advanced models incorporate limb darkening [19, 20].

Conclusion

This paper studies the darkening of a scene during a total solar eclipse using elementary mathematics. The decreasing luminances are obtained from camera pixel data. The theoretical model assumes uniform radiance from all regions of the Sun and does not require any use of calculus. Therefore, the results can be presented to introductory students with a knowledge of geometry and trigonometry. The analysis is rich in interdisciplinary connections involving astronomy (eclipses), physics (radiance), biology (luminance), psychology (perception of brightness), and computer science (RGB pixel color). The deviation from experiment and theory reveals the subtle concept of solar limb darkening in astronomy. For students in elementary grades and for the general public, the teacher can show the video [3] and the images of figure 1. Students of all ages will find the video and associated images fascinating.

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